V. *On the Magnetic Character of the Armour-plated Ships of the Royal Navy, and on the Effect on the Compass of particular arrangements of Iron in a Ship.*

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Received March 9,—Read March 16, 1865.

THE present paper may be considered as a sequel to a paper published in the Philosophical Transactions for 1860, page 337, under the title "Reduction and Discussion of the Deviations of the Compass observed on board of all the Iron-built Ships, and a selection of the Wood-built Steam-ships in Her Majesty's Navy, and the Iron Steam-ship 'Great Eastern'; being a Report to the Hydrographer of the Admiralty. By F. J. EVANS, Master R.N." Like the former, the present paper is presented to the Royal Society, with the sanction of the Lords Commissioners of the Admiralty.

In the brief interval which has elapsed since the publication of that paper, changes of the greatest importance have taken place in the construction of vessels of war, which have been accompanied by corresponding changes in the magnetic disturbance of their compasses. Not only has there been a great increase in the surface and mass of iron used in the construction of those parts of the ship in which iron was formerly used, but iron has been adopted for many purposes for which it was not then used, and much of the iron thus added far exceeds in thickness any that was formerly in use. Among the masses thus added we may specially mention iron masts and yards, armour-plating, and gun-turrets.

These changes have materially affected the problem of the correction of the deviation of the compass. They have not only greatly increased those errors which were formerly taken into account, but they have given importance to errors and causes of error which it was formerly considered might be safely neglected. These changes led to, if they did not necessitate, a complete revision of the mathematical theory of the deviations of the compass, and of the practical methods of ascertaining and applying the deviation.

This revision was undertaken by us at the request of the Admiralty, and the results are contained in the 'Admiralty Manual for ascertaining and applying the Deviations of the Compass caused by the Iron in a Ship,' published by the order of the Lords Commissioners of the Admiralty. London: POTTER, 1862. Second edition, 1863. It is gratifying to us to be able to state, as an indication that this work has been found

useful by others engaged in the like investigations, that it has been already translated into Russian, French, and German.

The methods of reduction previously in use, and which are those made use of in the paper already referred to, as well as in the valuable Reports of the Liverpool Compass Committee, are those deduced from the approximate formula for the deviation,

$$\delta = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta',$$

as given in the Supplement to the 'Practical Rules for ascertaining the Deviations of the Compass which are caused by the Ship's Iron,' published by the Admiralty in 1855.

In connexion with this formula use was made of the invaluable graphic method known as NAPIER'S curve.

At that time observations of horizontal and vertical force did not enter into the usual routine of observations made on board ship, although many very valuable observations of these forces had been made by the Liverpool Compass Committee; and no formulæ had been published for the deduction from such observations of any of the parts of the deviation. This will explain why, in the paper of 1860, the discussion was confined to the coefficients which are derived from observations of deviation only, viz. A, B, C, D, E.

The new modes of construction brought into prominence the diminution of mean directive force which a compass-needle suffers in an iron ship, particularly when placed between two iron decks. It is well known that in the interior of a thick iron shell the effect of the earth's magnetic force is nearly insensible. This is not caused by the iron of the shell *intercepting* the earth's magnetism, but by an opposite magnetism being induced which nearly neutralizes the earth's magnetism whatever be the inductive capacity of the shell, and whatever be the thickness of the shell, provided only that the thickness bears a considerable proportion to the diameter of the shell. When the shell is thin, the diminution of force is still considerable, but it then depends in a very much greater degree on the inductive capacity and the thickness of the shell. The destruction of force is total in the case of a spherical shell whatever be its thickness, if the inductive capacity be infinite.

An iron ship, as regards a compass-needle between decks, may be compared to a thin iron shell. Before the ship is launched, and when every particle of iron in her structure has by continued hammering become saturated with magnetism, she may be compared to a thin shell of high inductive capacity, and the directive force on a needle in the interior is consequently greatly diminished. When the ship is launched and placed successively on every azimuth, she may be compared to a thin shell of low inductive capacity. The mean directive force on a needle in her interior will be considerably diminished, but the diminution will depend much more on the thickness of the surrounding iron.

This diminution has been found so considerable in the case of iron-built and particularly iron-plated ships, as to have become a matter of serious consideration in selecting a place for the compasses.

Observations of horizontal force, for the purpose of ascertaining the diminution of the mean directive force, have now become part of the regular series of observations made in ships in which its determination is of importance, and formulæ and graphic methods, for the purpose of deducing from them the proportion of the mean value of the directive force to North to the earth's horizontal force, are given in the 'Admiralty Manual.'

Another error of the greatest importance, which has been brought into prominence in the modern class of iron-built ships, is the "heeling error."

The deviations obtained by the usual process of swinging are for a vessel in an upright position. It is found by experience that, as the vessel heels over, the north end of the compass-needle is drawn either to the weather or lee side, generally in the northern hemisphere to the former, and the deviation so produced when the ship's head is near North or South, often exceeds the angle of heel. This not only produces a deviation which may cause a serious error in the ship's course, but if the ship is rolling, and particularly if the period of each roll approximates to the period of oscillation of the compass, it produces a swinging of the compass-needle which may make the compass for the time useless for steering.

This error had been known to exist, and its amount had even been measured in the case of Her Majesty's ships Recruit (1846), Bloodhound (1847), Sharpshooter (1848), and in various cases recorded by the Liverpool Compass Committee (1855–61); but no method had been proposed for determining this error by observations made with the ship upright, and considerable obscurity was even supposed to rest on the causes and law of this deviation. The application of POISSON's formulæ has entirely removed the obscurity, and furnishes an easy method of determining the heeling error by observations of vertical force made on one or more directions of the ship's head. These observations have likewise now become a regular part of the complete series of magnetic observations made in the principal iron ships of Her Majesty's Navy.

Fortunately the mechanical correction of this error, when its amount is ascertained, is not difficult, and as the correction does not affect the deviation when the ship is upright, its application is free from some of the objections which exist to the mechanical correction of the ordinary deviation.

The importance of being thus able to detect the heeling error by observations of a simple kind made with the ship upright is great, and this is perhaps one of the most practically useful of the immediate results of the application of mathematical formulæ to this subject.

Besides these, which may be called the direct results of the additional observations now made, and of the application to them of the mathematical formulæ, there are some other results of the use of the formulæ which have a practical value as well as a theoretical interest.

Among these is the separation into their constituent parts of the several coefficients, so as to indicate the particular arrangements of the iron from which each arises. This is not only of great theoretical interest, but is of considerable practical importance in

indicating the place which should be selected for the compass, and also in enabling us to anticipate or account for the subsequent changes which take place in the deviation.

Another and perhaps even more important result is that we are enabled by observations made with the ship's head in one direction, and therefore when she is in dock or even on the stocks, to determine the coefficients and construct a table of deviations, including the heeling error, without swinging the ship. To explain this, we may observe that for the complete determination of the deviations of the compass when the ship is upright and in one geographical position, six coefficients are required. But of these two vanish when the iron is symmetrically arranged, two more are so nearly the same in ships of the same class that they can be estimated with a near approximation to the truth; we have therefore only two coefficients left, and these can be determined by an observation of deviation, and an observation of horizontal force made without altering the direction of the ship's head.

So as regards the heeling error, to determine this three additional quantities are generally necessary, but of these one is zero when the iron is symmetrically arranged; another may be estimated, and the third may then be determined by a single observation of vertical force.

The quantities so estimated change little after the ship is completed, so that any assumption made as to their value may be checked by subsequent observations.

These considerations will show the importance of not only making the observations we have mentioned, but of reducing the observations made, and of tabulating, discussing, and publishing the results of the observations. In the Tables it will be seen that the original observations are not given; they, as well as the curves and computations by which the coefficients are derived, are carefully preserved among the records of the Admiralty Hydrographic Office, and may at any time be referred to; but the coefficients, at least so far as regards the deviation of the horizontal needle, represent so exactly the observations made, that to give them here at length would be an unnecessary waste of space.

The observations, the results of which are tabulated, were made in the following manner. The deviations of the Standard Compass were observed by reciprocal simultaneous bearings of the Standard Compass and an azimuth compass on shore, in the manner described in the 'Admiralty Manual.' The admirable construction of the Admiralty Standard Compass, as regards design and workmanship, accuracy of adjustment and magnetic power, leaves nothing further to be desired for such observations. The arrangement of its four needles obviates, as we have shown in a former paper*, the sextantal error caused by the length of the needle when acted on by iron placed near it.

The deviations of the steering and maindeck compasses were obtained by observations of the direction of the ship's head by those compasses, made simultaneously with the observations of the Standard Compass. These compasses in the Royal Navy are of

* Philosophical Transactions, Part II. 1862.

simpler construction than the Standard, not being fitted with the azimuth circle, and generally having only two needles, but they are of little inferior accuracy, magnetic power and delicacy. The two needles are arranged so as to obviate the sextantal error above alluded to.

The Tables of deviations of these compasses have in all cases been most satisfactory, and on those points on which the directive force is very much diminished, they continue to give satisfactory indications which compasses of inferior workmanship would wholly fail to do.

The observations of horizontal force were made by vibrating a small flat lenticular needle $2\frac{3}{4}$ inches long and $\frac{1}{3}$ inch broad, fitted with a sapphire cap, on a pivot of its own, made to screw into the socket of the pivot of the Standard Compass, and comparing the time of vibration with that of the same needle vibrated on shore.

The observations of vertical force were made by vibrating a dipping-needle of $2\frac{3}{4}$ inches, placed in the position of the compass, the needle being made to vibrate in a vertical plane at right angles to the magnetic meridian. The observation might of course be made by vibrating the needle in the plane of the meridian and observing the dip; and in low dips that method is probably the best. In so high a dip as that of England, vibrations in the east and west plane are sufficiently accurate, and enable us to dispense with observations of dip.

In the selection of these instruments it has been found of great importance that they should be light, portable, easily and quickly fixed in position, capable of being placed in the exact position of the compass, should admit of observations being made quickly and in rough and boisterous weather, and should be such that each separate observation should give a useful result.

When the observer can command favourable circumstances of observation, as in the case of observations made in a ship on the stocks, it is possible that instruments of greater nicety may give more exact results, but for the ordinary observations which can be made in the process of swinging a ship, we have every reason to be satisfied with the results obtained from the instruments we have described.

As the formulæ made use of in the reductions are nowhere published except in the 'Admiralty Manual,' it seems necessary here to give them with a brief indication of the manner in which they are obtained.

The effect of the iron of a ship on the compass-needle is assumed to be due partly to the transient magnetism induced in the soft iron by the magnetism of the earth, and partly to the permanent magnetism of the hard iron. Simple physical considerations show that the components of the first in any three directions in the ship are linear functions of the components of the earth's magnetism in the same directions, the last is expressed by constant forces acting in the same three directions.

If, therefore, the components of the earth's force on the compass be X in the direc-

tion of the ship's head, Y to starboard, Z vertically downwards or to nadir, and if the components of the ship's permanent magnetism in the same directions be P, Q, and R, and of the total force of earth and ship in the same three directions X′, Y′, Z′, then

$$\text{Ship's force to head} \quad = X' - X = aX + bY + cZ + P, \quad \ldots \quad (1)$$

$$\text{Ship's force to starboard} = Y' - Y = dX + eY + fZ + Q, \quad \ldots \quad (2)$$

$$\text{Ship's force to nadir} \quad = Z' - Z = gX + hY + kZ + R, \quad \ldots \quad (3)$$

a, b, c, d, e, f, g, h, k being coefficients depending on the amount and arrangement of the soft iron of the ship. These are POISSON's fundamental equations, first given in the Mémoires de l'Institut, tom. v. p. 533.

To adapt these formulæ to observation, let

H be the earth's horizontal force,
ζ the easterly azimuth of the ship's head measured from the correct magnetic north;
θ the dip.

Then $X = H\cos\zeta$, $Y = -H\sin\zeta$, $Z = H\tan\theta$.

Substituting these values, and dividing (1) and (2) by H, *i. e.* taking the earth's horizontal force at the place as unit, equations (1) and (2) become

$$\text{Ship's force to head} \quad = \frac{X'-X}{H} = a\cos\zeta - b\sin\zeta + c\tan\theta + \frac{P}{H}. \quad \ldots \quad (4)$$

$$\text{Ship's force to starboard} = \frac{Y'-Y}{H} = d\cos\zeta - e\sin\zeta + f\tan\theta + \frac{Q}{H}. \quad \ldots \quad (5)$$

Dividing (3) by Z, *i. e.* taking the earth's vertical force as unit, we have

$$\text{Force of earth and ship to nadir} = \frac{Z'}{Z} = \frac{g}{\tan\theta}\cos\zeta - \frac{h}{\tan\theta}\sin\zeta + 1 + k + \frac{R}{Z}. \quad \ldots \quad (6)$$

If we resolve the forces (4) and (5) in the direction of the magnetic north, we shall find, besides periodical terms, one non-periodical term $\frac{a+e}{2}$, which therefore represents the mean force of the ship to North, and therefore $\left(1+\frac{a+e}{2}\right)H = \lambda H$, is the "mean force to North," or the mean value of the northern component of the force of earth and ship.

If we take the "mean force to North,' or λH for unit, or, in other words, divide by λH, we derive from (4) and (5) the following expressions for the force of earth and ship to North and to East respectively, viz.

$$\text{To North} = \frac{H'\cos\delta}{\lambda H} = 1 + \mathfrak{B}\cos\zeta - \mathfrak{C}\sin\zeta + \mathfrak{D}\cos 2\zeta - \mathfrak{E}\sin 2\zeta, \quad \ldots \quad (7)$$

$$\text{To East} = \frac{H'\sin\delta}{\lambda H} = \mathfrak{A} + \mathfrak{B}\sin\zeta + \mathfrak{C}\cos\zeta + \mathfrak{D}\sin 2\zeta + \mathfrak{E}\cos 2\zeta, \quad \ldots \quad (8)$$

in which H′ is the directive force of earth and ship on the needle, δ the deviation.

$$\lambda = 1+\frac{a+e}{2}, \quad \mathfrak{A}=\frac{d-b}{2\lambda}, \quad \mathfrak{B}=\frac{1}{\lambda}\left(c \tan\theta+\frac{P}{H}\right),$$

$$\mathfrak{D}= \frac{a-e}{2\lambda}, \quad \mathfrak{E}=\frac{d+b}{2\lambda}, \quad \mathfrak{C}=\frac{1}{\lambda}\left(f \tan\theta+\frac{Q}{H}\right).$$

From equations (7) and (8) we obtain

$$\tan\delta=\frac{\mathfrak{A}+\mathfrak{B}\sin\zeta+\mathfrak{C}\cos\zeta+\mathfrak{D}\sin 2\zeta+\mathfrak{E}\cos 2\zeta}{1+\mathfrak{B}\cos\zeta-\mathfrak{C}\sin\zeta+\mathfrak{D}\cos 2\zeta-\mathfrak{E}\sin 2\zeta}, \quad \ldots \ldots \quad (9)$$

whence if ζ' be the azimuth of the ship's head measured from the direction of the disturbed needle so that $\zeta'=\zeta-\delta$,

$$\sin\delta=\mathfrak{A}\cos\delta+\mathfrak{B}\sin\zeta'+\mathfrak{C}\cos\zeta'+\mathfrak{D}\sin(2\zeta'+\delta)+\mathfrak{E}\cos(2\zeta'+\delta). \quad \ldots \quad (10)$$

If the deviations are small, we have approximately

$$\delta=A+B\sin\zeta'+C\cos\zeta'+D\sin 2\zeta'+E\cos 2\zeta', \quad \ldots \ldots \quad (11)$$

in which A, B, C, D, E are (nearly) the arcs of which $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ are the sines.

The term $\mathfrak{B}\sin\zeta'+\mathfrak{C}\cos\zeta'$ may be put under the form $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}\sin(\zeta'+\alpha)$, in which α, called the starboard angle, is an auxiliary angle such that $\tan\alpha=\frac{\mathfrak{C}}{\mathfrak{B}}$.

If the soft iron of the ship be symmetrically arranged on each side of the fore-and-aft line of the ship through the compass, then

$$\begin{array}{lll} b=0, & d=0, & f=0, \\ \mathfrak{A}=0, & \mathfrak{E}=0, & \\ A=0, & E=0. & \end{array}$$

If we put $\mu=1+k+\frac{R}{Z}$, the expression of the nadir force of earth and ship in terms of earth's vertical force as unit, is

$$\text{Nadir force} = \frac{Z'}{Z}=\frac{g}{\tan\theta}\cos\zeta-\frac{h}{\tan\theta}\sin\zeta+\mu, \quad \ldots \ldots \quad (12)$$

If the ship heels over to starboard an angle i, $\mathfrak{B}$ and $\mathfrak{D}$ (or B and D) remain unaltered; and representing the altered values of $\mathfrak{A}$, $\mathfrak{C}$ and $\mathfrak{E}$ by $\mathfrak{A}_i$, $\mathfrak{C}_i$, and $\mathfrak{E}_i$, we have

$$\mathfrak{A}_i=\mathfrak{A}-\frac{g-c}{2\lambda}i,$$

$$\mathfrak{E}_i=\mathfrak{E}-\frac{g+c}{2\lambda}i,$$

$$\begin{aligned}\mathfrak{C}_i&=\mathfrak{C}-\left(\mathfrak{D}+\frac{\mu}{\lambda}-1\right)\tan\theta\, i\\ &=\mathfrak{C}-\chi i.\end{aligned}$$

The alteration in $\mathfrak{A}$ and $\mathfrak{E}$ may generally be neglected; that in $\mathfrak{C}$ is often of great importance. The quantity $\chi=\left(\mathfrak{D}+\frac{\mu}{\lambda}-1\right)\tan\theta$ is called the heeling coefficient, and represents the degrees of deviation to windward, or the high side of the ship, produced by a heel of one degree when the ship's head is North or South by the disturbed compass.

The effect of the coefficients on the deviation is most easily seen by considering the effect of the derivative coefficients λ, $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, and of the heeling coefficients, which, for convenience of reference, are here arranged in a tabular form. These are as follows:—

$\lambda=1+\frac{a+e}{2}$ is a factor generally less than 1, giving the northern component of the mean directive force on the needle, or "mean force to North."

$\mathfrak{A}=\frac{d-b}{2\lambda}$ (approximate value in degrees $=$A) is the constant term of the deviation; its real value is 0 when the iron is symmetrically placed on each side of the compass, and it is not in general distinguishable from an index error of the compass, or an error in the assumed variation of the compass (declination).

$\mathfrak{B}=\frac{1}{\lambda}\left(c\tan\theta+\frac{\mathrm{P}}{\mathrm{H}}\right)$ (approximate value in degrees $=$B) is the maximum of semicircular deviation from fore-and-aft forces; $\frac{c}{\lambda}\tan\theta$ arises from soft iron; $\frac{\mathrm{P}}{\lambda\mathrm{H}}$ from hard iron.

$\mathfrak{C}=\frac{1}{\lambda}\left(f\tan\theta+\frac{\mathrm{Q}}{\mathrm{H}}\right)$ (approximate value in degrees $=$C) is the maximum of semicircular deviation from transverse forces; $\frac{f}{\lambda}\tan\theta$ arises from soft iron, and is zero if the iron is symmetrically arranged; $\frac{\mathrm{Q}}{\lambda\mathrm{H}}$ from hard iron.

$\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$ (approximate value in degrees $=\sqrt{\mathrm{B}^2+\mathrm{C}^2}$) is the maximum of semicircular deviation.

$\frac{\mathfrak{C}}{\mathfrak{B}}$ is the tangent starboard angle, or of angle measured to right of fore and aft of line of ship, in which the force causing the semicircular deviation acts.

$\mathfrak{D}=\frac{a-e}{2\lambda}$ (approximate value in degrees $=$D) is the maximum of quadrantal deviation from soft iron symmetrically placed.

$\frac{\mathfrak{D}}{2}-\frac{1}{2}\left(\frac{1}{\lambda}-1\right)=\frac{a}{2\lambda}$ is the part of $\mathfrak{D}$ arising from fore-and-aft soft iron.

$\frac{\mathfrak{D}}{2}+\frac{1}{2}\left(\frac{1}{\lambda}-1\right)=-\frac{e}{2\lambda}$ is the part of $\mathfrak{D}$ arising from transverse soft iron.

$\mathfrak{E}=\frac{d+b}{2\lambda}$ (approximate value in degrees $=$E) is the maximum of quadrantal deviation from soft iron unsymmetrically placed.

$\left(\mathfrak{D}+\frac{\mu}{\lambda}-1\right)\tan\theta=\chi$ is the heeling coefficient, or the deviation to windward in degrees for one degree of heel when ship's head North or South by disturbed compass.

$\left(\mathfrak{D}+\frac{1}{\lambda}-1\right)\tan\theta$ is the part of heeling coefficient from transverse soft iron.

$\left(\frac{\mu}{\lambda}-\frac{1}{\lambda}\right)\tan\theta$ is the part of heeling coefficient from vertical soft iron, and vertical force of hard iron.

$\frac{g}{\tan\theta}$ is the increase or decrease of vertical force above or below mean when ship's head is North or South.

$$\frac{P}{\lambda}+\frac{c}{\lambda}\text{H}\tan\theta=\mathfrak{B}\text{H},$$

$$\frac{P}{\lambda}+\frac{c}{\lambda}\text{H}'\tan\theta'=\mathfrak{B}'\text{H}'$$

are the equations for determining c and P separately when $\mathfrak{B}$ has been determined in two different latitudes;

$$\frac{c}{\lambda}=\frac{\mathfrak{A}_i-\mathfrak{A}_{i'}}{i-i'}-\frac{\mathfrak{E}_i-\mathfrak{E}_{i'}}{i-i'},$$

$$\frac{P}{\lambda}=\mathfrak{B}\text{H}-\frac{c}{\lambda}\text{H}\tan\theta$$

are equations for determining c and P separately when observations have been made in one geographical position, but on two different angles of heel;

$$\mathfrak{B}=\frac{1}{\lambda}\frac{\text{H}'}{\text{H}}\cos\zeta'-(1+\mathfrak{D})\cos\zeta,$$

$$\mathfrak{C}=-\frac{1}{\lambda}\frac{\text{H}'}{\text{H}}\sin\zeta'+(1-\mathfrak{D})\sin\zeta$$

are equations for determining $\mathfrak{B}$ and $\mathfrak{C}$ by observations of deviation and horizontal force on one azimuth of the ship's head, λ and $\mathfrak{D}$ being known or estimated.

There is a physical representation of POISSON'S fundamental equations so simple, and which gives us so great a power of estimating the effect on the compass of different arrangements of iron in a ship, as well as of tracing to their cause any peculiarities in the observed deviation, that it seems desirable, before entering on the peculiarities of structure and deviation in armour-plated ships, to explain this representation, and to show how it explains the phenomena of deviation.

If an infinitely thin straight rod of soft iron be magnetized by the induction of the earth, the effect will be the same as if each end became a pole having an intensity proportional to the component of the earth's force resolved in the direction of the rod, and to the section and capacity for induction of the rod.

Let us now suppose nine soft iron rods placed as Plate X. It will be seen that for each rod we must distinguish the two cases, that in which its coefficient is $+$, and that in which it is $-$. It will also be seen that in the three cases, viz. $-a$, $-e$, $-k$, in which the rod passes through the compass, we may consider both ends as acting, but that in other cases it is convenient to consider only the action of the near end, and that the far end is at an infinite distance.

The rod a, it will be observed, can only be magnetized by the component X, b only by Y, and c only by Z; and if we call aX, bY, and cZ the force with which these rods attract the north end of the needle, and if we suppose, as we are at liberty to do, the

rods being imaginary, that they exercise no action on one another, a, b, and c will produce a force to head

$$=aX+bY+cZ;$$

so d, e, and f will produce a force to starboard

$$=dX+eY+fZ,$$

and g, h, and k will produce a force to nadir

$$=gX+hY+kZ.$$

By comparing these results with POISSON'S formulæ, we see that for the effect of the soft iron of the ship, however complicated its arrangement may be, we may substitute the nine soft iron rods.

The quantities P, Q, R in the general equations may be conveniently represented by three bar-magnets, placed in fixed positions in the ship; P attracting the north end of the compass-needle to the head, Q to starboard, and R to nadir.

Very simple considerations will show us that the two rods a and e will increase the directive power on the needle in the proportion of $1+\frac{a+e}{2}:1$, and that the other seven rods, as well as the permanent forces P, Q, R, will not affect the mean directive force.

Simple considerations will also show that a and e will produce a deviation,

$$\frac{a-e}{2\lambda}\sin 2\zeta'=\mathrm{D}\sin 2\zeta'$$

nearly. Like considerations will show that c and P will produce a deviation,

$$\frac{c\mathrm{Z}+\mathrm{P}}{\lambda\mathrm{H}}\sin\zeta'=\left(\frac{c}{\lambda}\tan\theta+\frac{\mathrm{P}}{\lambda\mathrm{H}}\right)\sin\zeta'=\mathrm{B}\sin\zeta'.$$

Also that f and Q will produce a deviation,

$$\frac{f\mathrm{Z}+\mathrm{Q}}{\lambda\mathrm{H}}\cos\zeta'=\left(\frac{f}{\lambda}\tan\theta+\frac{\mathrm{Q}}{\mathrm{H}}\right)\cos\zeta'=\mathrm{C}\cos\zeta'.$$

The other less important terms, as well as the heeling error, may be obtained in the same manner.

DISCUSSION OF THE TABLES.

At the risk of some repetition it may be convenient to give here a brief explanation of the quantities tabulated.

The first five quantities, A, B, C, D, E, are the "approximate coefficients" which give the deviation of the compass on every course by means of the expression

$$\delta=\mathrm{A}+\mathrm{B}\sin\zeta'+\mathrm{C}\cos\zeta'+\mathrm{D}\sin 2\zeta'+\mathrm{E}\cos 2\zeta',$$

in which δ is the deviation, ζ' the azimuth of the ship's head measured eastward from the direction of the disturbed needle, A, B, C, D, E being expressed in degrees and minutes.

This expression is sufficiently accurate for deviations not exceeding 20°; for larger deviations, the exact expression for the deviation given in the preceding part of the

paper requires the use of the "exact coefficients" $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, which are not expressed in degrees and minutes, but are nearly the sines of the corresponding angles A, B, C, D, E.

For the purpose of this discussion we may confine our attention to A, B, C, D, E.

A is the "constant part of the deviation." A *real* value of A can only be caused by elongated horizontal masses of *soft* iron *unsymmetrically* arranged with reference to the compass, and would be the same in all parts of the globe. An arrangement of horizontal soft iron rods such as that in fig. 1 would give a positive value to A and no other term in the deviation. This, however, is not an arrangement which would occur on shipboard.

Fig. 1.

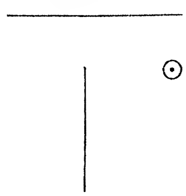
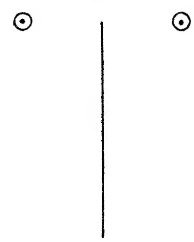


Fig. 2.



A soft iron rod such as that in fig. 2 would give $+A$ to the starboard compass, combined with $+E$; and $-A$, combined with $-E$, to the port compass.

This arrangement is not unfrequent in the relative positions of the spindle of the steering-wheel and the binnacle compasses placed near it for the guidance of the helmsman.

In compasses placed in the middle line of the ship such an arrangement is improbable, and in such case A has probably little or no *real* value. An *apparent* value may, however, be given to A by index-error in the compass on board, index or other error in the shore compass with which it is compared, or error of observations generally.

When the ship heels over, an elongated horizontal mass of iron, which was symmetrically placed from being below the compass, as the screw-shaft or the keel, is thrown to one side, and an A may then be introduced caused by and proportional to the angle of heel; but this has not been found of sufficient amount to require attention in practice.

The terms $B \sin \zeta' + C \cos \zeta'$ make up together what is called the "semicircular deviation;" B depending on fore-and-aft forces, and having its zero when the ship's head is North or South, its maximum when it is East or West; C depending on transverse forces, and having its zero when the ship's head is East or West, its maximum when it is North or South.

B consists of two parts, one a coefficient arising from vertical induction in soft iron before or abaft the compass, and being multiplied by the tangent of the dip and a factor $\frac{1}{\lambda}$ hereafter explained; the other a coefficient arising from permanent magnetism of the

hard iron in the ship acting in the fore-and-aft line, and multiplied by the reciprocal of the earth's horizontal force, and also by the factor $\frac{1}{\lambda}$. The last part may be considered as itself consisting of two parts; one, of the subpermanent magnetism induced while the ship was building by the vertical component of the earth's force, and which probably bears some relation to the transient magnetism induced by the same vertical component; another, of the subpermanent magnetism induced while the ship was building by the headward component of the earth's horizontal force.

C theoretically consists of similar parts acting towards the sides of the ship; but as the iron may in general be considered as symmetrically arranged on each side of the compass, the value of C is probably, in all cases when the ship is upright and the compass is amidships, to be attributed to subpermanent magnetism induced while the ship was building by the transverse component of the earth's horizontal force. The part of B consisting of transient induced magnetism varies as the tangent of the dip. The other part of B and C vary inversely as the earth's horizontal force. As regards changes which take place after launching, without a change of geographical position, there are differences between the several parts of B and C which require notice.

When the ship is launched, notwithstanding that her head is no longer kept in one fixed direction, the forces which cause the two first-mentioned parts of B still act in precisely the same direction as before, and these two parts probably undergo little change.

With the third part of B and the whole of C the case is very different. The forces which cause these parts cease to act in the same direction as at first. If the vessel is allowed to swing at her anchors, or is under sail or steam, she will probably on an average be nearly as much on one point as on another; or, which would come to nearly the same thing, if she is lying in a tideway she may be alternately for six hours in one direction and for six hours in the opposite direction. A great portion of the C and of that part of the B which arose from horizontal force thus become dispelled.

The symmetry which gives C its character ceases the moment the ship heels. An addition is then made to C proportional to the angle of heel, and this addition consists in fact of two parts, corresponding to the two parts of B which, as we have seen, do not exist in the original C, viz. a part consisting of transient magnetism induced by the vertical force, and a part consisting of subpermanent magnetism induced by the same force. These will be more conveniently considered when we come to discuss the heeling error.

The semicircular deviation may be put under the form $\sqrt{B^2+C^2}\sin(\zeta'+\alpha)$, in which $\sqrt{B^2+C^2}$ represents the maximum of semicircular deviation, α $\left(\tan\alpha=\frac{C}{B}\right)$ the angle to the right of the ship's head of the force causing this deviation; for convenience, these two quantities are tabulated in the eleventh and thirteenth columns.

The terms $D\sin 2\zeta'+E\cos 2\zeta'$ make up what is called the "quadrantal deviation."

This can only be caused by *horizontal* induction in soft iron. E can only be caused by horizontal induction in soft iron *unsymmetrically* distributed, but of any shape; an E may therefore be caused by the compass being placed out of the midship line and exposed to the influence of spherical or cylindrical masses, such as the iron gun-turrets of modern war-vessels.

D, which in ordinary cases is always $+$, is caused by horizontal induction in soft iron arranged according to one or other of the following types:—

Fig. 3.

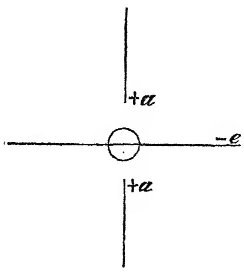
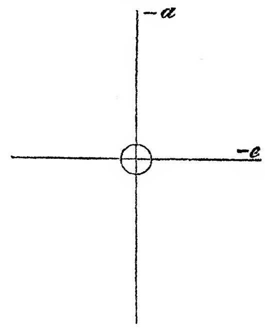


Fig. 4.



In the figures $+a$ represents masses of soft iron entirely before or entirely abaft the compass, as engines, boilers, funnels, iron masts, &c.; $-a$ represents soft iron extending through the position of the compass, as the keel and hull of the ship, the screw-shaft, armour-plating, &c., the effect of the latter in almost all cases exceeding that of the former, so that a is in general negative; $-e$ represents the effect of all the transverse soft iron, as the bottom of the ship, the iron decks (except where interrupted by hatchways near the compass), iron deck beams, and the engines, boilers, &c.; $+e$ represents the masses of iron, comparatively few in number, which lie to one side of the compass, as decks where the compass is in or over a hatchway, occasional guns, davits, &c. In every ship which has been examined, the effect of the transverse iron extending through the position of the compass exceeds that of any masses of iron wholly on one side, and e is negative and greater than a; and as $\mathfrak{D}=\frac{a-e}{2\lambda}$, $\mathfrak{D}$, and consequently D, are in almost all cases $+$.

D and E do not change with a change of geographical position.

In almost all cases in iron-built ships, not only is the direction of the needle directly affected by the iron of the ship, but a further prejudicial effect is caused by the soft iron diminishing the mean directive force of the needle, and so indirectly increasing the effect of all disturbing forces. This is shown by the factor λ, which gives the mean value of the directive force, or rather of the northern component of the directive force in the ship, and which is almost always less than unity, the force on shore being considered as unity.

The cause of this diminution will be seen by figs. 3 & 4. In fig. 4 a little consideration will show that both $-a$ and $-e$ diminish the directive force. In fig. 3 $+a$ in-

creases the directive force, $-e$ diminishes it; but as $-e$ always exceeds $+a$, the result is a diminution on the whole.

The expression for λ in terms of a and e is

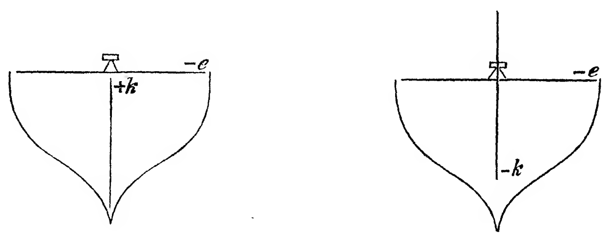
$$\lambda=1+\frac{a+e}{2}.$$

The tabulated values of λ are obtained by comparing the terms of vibration of a horizontal needle vibrated in the position of the compass in the ship and also on shore; λ does not change with a change of geographical position.

The determination of $\mathfrak{D}$ and λ gives us the means of determining the two parts a and e, and also the two parts of which D is composed, separately; and these are accordingly tabulated.

The preceding are the only coefficients which affect the compass when the ship is upright; but when the ship heels over, new disturbing forces are called into play, caused by arrangements of soft or hard iron of one or other of the following types:—

Fig. 5.



$-e$ represents, as before, the transverse soft iron, which will evidently, as the ship heels over, produce a force to windward, or the high side of the ship, on the north end of the needle. If the rods $+k$ and $-k$ represent soft iron, then $+k$ gives a force acting downwards on the north end of the needle, which, as the ship heels, becomes a force to windward; $-k$ a force acting upwards, which, as the ship heels, becomes a force to leeward. The permanent magnetism of the ship will generally act downwards if the compass is over the end which has been South in building, upwards if over the end which has been North in building. The amount of the two forces may be ascertained by vibrating a dipping-needle on shore and in the ship with her head in certain positions. The proportion of the mean vertical force on board to the vertical force on shore is denoted by the coefficient μ, which is tabulated for those ships in which the observations have been made.

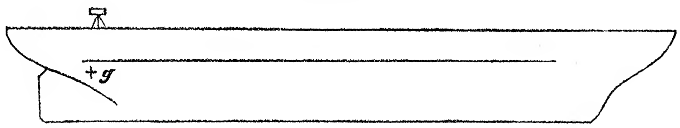
From the values of μ, $\mathfrak{D}$ and λ we obtain by a simple formula, viz. $\left(\mathfrak{D}+\frac{\mu}{\lambda}-1\right)\tan\theta\,1°$, the "heeling coefficient to windward," or the deviation to windward caused, when the ship's head is N. or S. by compass, by an angle of heel of 1°. When this coefficient has a negative sign it indicates a deviation to leeward. The values of the heeling coefficient so deduced are tabulated. The value changes with a change of geographical position.

From the values of μ, $\mathfrak{D}$ and λ we may also determine how much of the heeling error arises from the transverse soft iron represented in the figures 3, 4 & 5, and how

much from the vertical soft iron and the hard iron, the first $=\left(\mathfrak{D}+\frac{1}{\lambda}-1\right)$ tan θ i°, the second $=\left(\frac{\mu}{\lambda}-\frac{1}{\lambda}\right)$ tan θ i°; and these two parts are tabulated in the next columns.

If we have not an opportunity of observing the vertical force on a sufficient number of points to obtain its mean value, the values observed will be affected by soft iron represented by the rod g, in the following figure:—

Fig. 6.



the value of μ on any azimuth ζ being in fact increased by $+\frac{g}{\tan\theta}\cos\zeta$, where θ is the dip. It is therefore convenient to know the values of g or $\frac{g}{\tan\theta}$, and these are also tabulated; g does not change with a change of geographical position.

In comparing the heeling error when the ship's head is North or South, we must beware of falling into the error of confusing the two senses in which these words may be used. It may seem most natural to suppose the ship's head to be North or South when upright, and that she is then heeled over without altering her direction. In that case we should have (nearly)

Heeling error head North : heeling error head South : : $1-\mathfrak{B}:1+\mathfrak{B}$.

In fact the heeling error is nearly inversely proportional to the directive force on the needle.

But this is not the sense in which the term is generally used. In general we suppose the ship swung when heeled to starboard and again when heeled to port, and the deviations tabulated in the usual way, according to the ship's azimuth by disturbed compass. In this case, which is the simplest mode of considering the error for the purpose of correction, the heeling error, head North, will only differ from the heeling error, head South, by reason of the quantity g, i. e. by reason of the difference of the vertical and not of the horizontal forces in the two positions.

The importance of the heeling error, owing to its large amount in certain ships, will be seen in the discussion of the values given in the Tables; and the importance of being able to determine it by observations easily made, and without the necessity of actually heeling over the ship, can hardly be overrated.

We are now in a position to consider the numerical values of the coefficients given in the Tables.

Constant Deviation.

A.

The values of A, when the compass is placed in the middle line of the ship, and when the deviations have been observed with every care, are always so small, that the values which appear in the Tables may be considered rather as errors of adjustment and observation than as real values. In fact it may be inferred that in all cases where the compass is in the middle line of the ship, we may consider A as zero. It results from this, and is important in practice, that we may safely take the mean of the compass bearings of any object, on four or more equidistant compass courses, as the *correct* magnetic bearing; observing, however, that if we observe on four points only, and D be large, these ought to be either the cardinal or the quadrantal points.

Semicircular Deviation,

$$B \sin \zeta' + C \cos \zeta'.$$

The points which require attention are,—

1. Its original value and its connexion with the direction of the ship in building, and the position of the compass in the ship.
2. The changes which take place after launching.
3. The subsequent changes.
4. The changes which take place on a change of geographical position.

1. In wood-built ships, as may be seen by an inspection of the Deviation Tables given in the work of the late Captain E. J. JOHNSON, R.N., on the deviation of the compass, the direction of the force causing the semicircular deviation is in northern latitudes nearly towards the ship's bow. In iron-built ships it is nearly to that part of the ship which was South in building; or, in other words, the starboard angle as given in the Tables, is nearly the same as the azimuth of the ship's head to the East of South in building; thus,—

	Direction of head in building.	Starboard angle, or direction of semicircular deviation.
Orontes . .	N. 66° W. or S. 246° E.	235°
Tamar . . .	West or S. 270° E.	279°

The case of the armour-plated ships is an interesting exception to this rule. Such ships are generally plated after launching, and in a different position from that of building. In these ships the angle of the semicircular force is generally intermediate between the angle of the ship's head to the East of South in building, and the like angle in being iron plated; thus,—

	Direction of head in building.		Direction of head in plating.		Direction of Semicircular Deviation.
Warrior . . .	N. 3° E. or	S. 177° E.	N.W. or	S. 225° E.	195°
Black Prince .	S. 20 E.	20	South.	0	8
Defence . . .	S. 47 W.	313	S. 19° E.	19	0
Resistance . . Valiant . . .	West.	270	generally to westward . . .		313 282

From these results we may infer that the process of plating an iron ship in the direction opposite to that of building will always produce a diminution, which in some cases may become a reversal of her semicircular deviation; and that by duly taking advantage of this circumstance, the deviations of iron-plated ships may be brought within manageable limits.

The Tables show, as might have been anticipated, the much larger amount of the deviation in the steering and main-deck compasses than in the Standard Compass, and the advantages to be derived from a judicious selection of a place for the compass; unfortunately even in the case of the Standard Compass the choice of position is so limited by the exigencies of the arrangements for working and fighting the ship, that the deviations in these compasses are generally larger than could be wished.

2. After launching, and when the vessel is swinging at anchor, or sailing or steaming in various directions, the values of B and C generally diminish rapidly; and this change would no doubt be accelerated by the vessel being exposed to blows or jars in a position different from that of building.

The following cases show a rapid change of B and C after launching. The most instructive have been selected from the Tables, but the elaborate series of observations made in the Great Eastern (Phil. Trans. 1860) are the most conclusive, as that ship was in every respect prepared for sea, and the observations are strictly comparable throughout.

H.M.S. Achilles, built in dry dock at Chatham, and fully plated there also, head S. 52° E., floated out of dock 24th December 1863, and moored head and stern in the River Medway, head S. 62° E. In March 1864, after taking in steam machinery, the ship made a short trial trip down the river, and then returned to the former moorings, but with her head secured in the opposite direction, or N. 62° W. Equipment and fittings completed by October 11th, when the head was shifted round to S. 55° E., and on the following day steamed to Sheerness and commenced sea service.

		𝔅.	ℭ.
1863.	Dec. 23.—In dock at Chatham	+·464	+·323
1864.	Sept. 26.—Complete for sea, head N. 62° W. . .	+·377	+·037
	Oct. 11.—Complete for sea, head S. 55° E. . .	+·355	+·062
	Oct. 13.—Swinging at anchor, Sheerness . . .	+·362	+·047
	Dec. 5.—At Plymouth, after 25 days in dock, head S. 79° E.	+·361	+·123

H.M.S. Royal Oak, wood-built ship, iron-plated in dock at Chatham, head S. 49° E.

1863.	Mar. 19.—Floated out of dock	+·253	+·287
	April 11.—Swinging at anchor, River Medway . .	+·231	+·197
	June 2.—Swinging at anchor, River Medway . .	+·248	+·128
1864.	Jan. 8.—Swinging at anchor, Plymouth . . .	+·218	+·172

The example of the Achilles is very instructive. The large value of ℭ+·323 giving

a C of 19°, which was caused by the ship having been built, plated, and moored with the starboard side South, is reduced to $+\cdot037$ or $2^\circ\ 10'$ by lying for six months with the port side South. This amount does not alter materially while the ship is allowed to swing, but when she is twenty-five days in dock with the starboard side South, it suddenly rises to $+\cdot123$ or 7°.

𝔅, it will be observed, changes much less at first, and hardly changes at all afterwards; this difference must be attributed in part to this, that while the whole of ℭ is to be attributed to subpermanent magnetism arising from horizontal induction in transverse hard iron, a large part of the original 𝔅 was probably caused by the transient magnetism arising from vertical induction in soft iron, and a further part by the subpermanent magnetism arising from vertical induction in hard iron, so that possibly not more than ·100 was caused by the subpermanent magnetism arising from induction from the headward component of the horizontal force, nearly the whole of which may have been removed by six months' reversal of her direction, so as to leave little room for subsequent change of 𝔅.

In connexion with this part of the subject we may observe that the same circumstances which cause the transient magnetism arising from horizontal induction in transverse iron $(-e)$ to be greater than the transient magnetism arising from horizontal induction in fore-and-aft iron $(-a)$, lead us to expect that the subpermanent magnetism arising from horizontal induction in transverse hard iron (ℭ) will be greater than the subpermanent magnetism arising from horizontal induction in fore-and-aft hard iron (changing part of 𝔅), and that consequently we should expect the relative changes of ℭ which take place on a change of direction to be greater than those of 𝔅, and this will be found to be verified in almost all cases, except when the ship has been built nearly North and South.

3. After a certain time, which may be roughly estimated at a year after launching, this process seems to stop, and the values of B and C remain remarkably permanent. The former paper* contains numerous examples of this in ordinary iron-built ships.

This will appear also from the following instances of the iron-plated ships.

			Standard Compass. 𝔅.	ℭ.
Warrior.	September	1861 . . .	$-\cdot449$	$-\cdot124$
	October	1861	$-\cdot409$	$-\cdot092$
	July	1862	$-\cdot321$	$-\cdot114$
	June	1863	$-\cdot317$	$-\cdot132$
	July	1864	$-\cdot311$	$-\cdot054$
	October	1864	$-\cdot307$	$-\cdot072$
Defence.	February	1862	$+\cdot464$	$+\cdot005$
	March	1863	$+\cdot379$	$-\cdot034$
	December	1863	$+\cdot403$	$-\cdot016$
	April	1864	$+\cdot391$	$-\cdot007$
	October	1864	$+\cdot379$	$-\cdot034$

* Philosophical Transactions, Part II. 1860.

		Standard Compass. 𝔅.	ℭ.
Black Prince.	November 1861	+·422	+·058
	September 1862	+·383	+·074
	July 1863	+·384	+·067
	April 1864	+·389	+·086
	October 1864	+·349	+·050
Resistance.	August 1862	+·149	−·158
	June 1863	+·152	−·138
	December 1863	+·106	−·120
	December 1864	+·065	−·153

It will be remembered in the foregoing examples that the ships have been frequently subjected to the strains in docking, trials, in gales of wind, and at high rates of speed, and especially to concussions from the drilling and firing their heavy ordnance.

A striking example of the permanency of the magnetism of an "old" iron ship after severe concussion is afforded in the case of the Adventure troop-ship built in 1854. This ship, in the course of foreign service during a fog, struck on a rock with sufficient force to tear away and crush in 20 feet of the stem and bow under water; appended are the coefficients observed *before* proceeding on the foreign service, and *after* the injuries sustained had been repaired in dock.

	𝔅.	ℭ.
1862. April 26th . . .	−·073	+·186
1862. October 28th . . .	−·071	+·186

An equally close agreement will be found, on reference to the Tables, to exist in the other magnetic coefficients of this ship; the exact accordance of the numerical values is of course accidental, but is conclusive as to the great wear and tear and rough usage an old iron ship can undergo without her magnetic conditions being changed.

4. The determination of the proportion of the semicircular deviation, or rather of B, which arises from vertical induction in soft iron, and that which arises from the permanent or subpermanent magnetism of hard iron, is a matter of great interest. Theoretically it may be determined in two modes, either by observing the deviation in two different magnetic latitudes, or by observing the deviation with the ship upright and heeled over. Unfortunately there is a great want of observations under these circumstances. The deviations of the iron-plated ships, given in the Tables, were carefully observed both at Lisbon and Gibraltar, but the difference of latitude between either place and England is too small, and the change in the subpermanent magnetism too great to enable us to derive any very certain results from these observations.

The difficulty of heeling a large ship is so great that few observations except in an upright position can be expected; we owe, however, to the zeal of the officers in command of the Warrior*, Black Prince, and Defence, that these ships were swung at

* Magnetic science is indebted to the Honourable Captain COCHRANE of Her Majesty's Ship Warrior, for the interest he has evinced, and the assistance he has rendered in obtaining complete records of that ship; and

Lisbon upright, and heeled about 7° to starboard and to port. The agreement of the values of the coefficient $\frac{c}{\lambda}$ derived by the different methods is not very satisfactory, and it can only be considered as a rough approximation to the truth.

From the equation for comparison of semicircular deviation in different latitudes

$$\frac{P}{\lambda}+H\tan\theta\frac{c}{\lambda}=\mathfrak{B}H.$$

	$\frac{P}{\lambda}$.	$\frac{c}{\lambda}$.
Warrior	$-\cdot471$	$+\cdot058$
Black Prince . . .	$+\cdot061$	$+\cdot142$
Defence	$+\cdot206$	$+\cdot079$
Resistance. . . .	$-\cdot330$	$+\cdot190$

From heeling-error formulæ.	$\frac{c}{\lambda}$.
Warrior	$+\cdot108$
Black Prince	$+\cdot181$
Defence	$+\cdot119$

Taking the mean of the several values in the ships.

	Original value of B.	$\frac{c}{\lambda}$.	Part of B from soft iron.	Part of B from hard iron.
Warrior	$-24\frac{1}{4}^{\circ}$	·083	$+12^{\circ}$	$-36\frac{1}{4}^{\circ}$
Black Prince	$+23$	·161	$+23$	0
Defence	$+25\frac{3}{4}$	·099	$+14\frac{1}{4}$	$+11\frac{1}{2}$

Taking the present values of B in the ships.

	B.	$\frac{c}{\lambda}$.	Part of B from soft iron.	Part of B from hard iron.
Warrior	-17°	·083	$+12^{\circ}$	-29°
Black Prince	$+19$	·161	$+23$	-4
Defence	$+21$	·099	$+14\frac{1}{4}$	$+6\frac{3}{4}$

And in any other magnetic latitude for which the horizontal force is H, the horizontal force in England being 1 and the dip θ, we should have

Warrior . . . . $B=-\frac{29^{\circ}}{H}+4\frac{3}{4}^{\circ}\tan\theta.$

Black Prince. . . $B=-\frac{4^{\circ}}{H}+9\frac{1}{4}^{\circ}\tan\theta.$

Defence . . . . $B=\frac{6\frac{3}{4}^{\circ}}{H}+5\frac{3}{4}^{\circ}\tan\theta.$

also to William Mayes, Esq., Master of Her Majesty's Ship Defence, for a valuable series of observations made in that ship, and for his exertions in obtaining results in several ships of the Channel Squadron.

Quadrantal Deviation, $D \sin 2\zeta' + E \cos 2\zeta'$.

Mean force to North λH.

The Tables show that the values of E when the ship is upright and the compass in the midship line, give no certain indication of any real value. The more accurate the instrument, and the more careful the observations, the smaller E generally is.

When the compass is not in the midship line the case is different; an E may then have a considerable value. Instances of this will be seen in the deviations of the Royal Sovereign, the peculiar construction and fittings of which ship made it necessary to place compasses considerably out of the midship line, and with gun turrets placed diagonally to them.

At the steering wheel on upper deck		$E = -9^\circ\ 14'$
At the steering wheel in captain's cabin		$E = -5\ 10$
Forward on lower deck	Port side	$E = +4\ 38$
	Starboard side	$E = -4\ 42$

It will easily be seen that a $+E$ would be caused by a gun turret in the first and third quadrant relatively to the compass, and a $-E$ by a turret in the second and fourth. The close agreement of the numerical value of E in the two last examples, with the difference in their signs, is striking.

The value of the E introduced by the ship heeling by an angle i to starboard being

$$-\frac{c+g}{2\lambda}i,$$

and both c and g being generally positive, we should expect a $-E$ when the ship heels to starboard, a $+E$ when she heels to port, and this is the case in the few instances we have in the Tables.

		E.
Warrior.—Standard Compass	$7\frac{1}{2}^\circ$ to starboard	$-0^\circ\ 45'$
	$7\frac{1}{2}$ to port	$+0\ 59$
Black Prince.—Standard Compass	$6\frac{1}{2}$ to starboard	$-1\ 25$
	$6\frac{1}{2}$ to port	$+1\ 50$
Defence.—Standard Compass	$7\frac{1}{2}$ to starboard	$-0\ 05$
	$7\frac{1}{2}$ to port	$+1\ 50$

D.

As regards D, the most important point is its magnitude in different positions in ships of different classes.

The usual or average value of D has greatly increased since the publication of the Paper in 1860. In that paper it was observed that a value for this coefficient not exceeding 4° and ranging between that amount and 2°, might be assumed to represent

the average or normal amount in vessels of all sizes, and in only two vessels mentioned in that paper did D exceed 5°.

In the iron-built armour-plated ships its average amount in the Standard Compass is about 7°, in the steering-compass about 10°, and in the main-deck compass about 12°. In the wood-built iron-plated ships the value of D is small.

The following Table gives the value in different ships.

	Warrior.	Black Prince.	Achilles.	Defence.	Resistance.	Hector.	Valiant.	Royal Oak (wood-built).
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
Standard compass	+ 8 27	+ 7 38	+ 6 58	+ 7 0	+ 6 17	+5 24	+4 54	+3 09
Starboard steering......	+11 56	+10 32	+ 8 51	+10 16	+ 8 28	+8 24	+6 52	+1 47
Main deck	+11 43	+13 16	+12 13	+14 35	+14 0	+9 47	+8 05	+1 28

The large amount in the Standard and Steering Compass of the Warrior is doubtless owing to the rifle tower which is immediately before them, and which gives a $+a$. The small comparative values in the Hector and Valiant to the iron-plating being extended from end to end in the ship giving a $-a$, and the absence of a complete transverse armour bulkhead, the existence of which in the Defence and Resistance, as well as in the Warrior and Black Prince, give large $-e$, and consequently large deviations in the binnacle and main-deck compasses.

Between the Resistance and the Defence there is a remarkable difference. These are nearly sister ships, but with this difference, that from the different position of the mizen-mast in the two ships their standard and steering compasses are very differently placed with reference to the transverse armour bulkhead. In the Resistance the Standard Compass is exactly above the bulkhead at a height of 12 feet. The steering-compass is about 4 feet in front, and the same height above it; while in the Defence these compasses are about 20 feet abaft it.

Such a bulkhead, when magnetized at right angles to its plane, will produce a fore-and-aft force on all points in, or nearly in, the same plane in the opposite direction to the magnetizing force. It will therefore, in the case of the standard and steering-compasses of the Resistance, introduce a $-a$ as well as a $-e$, while it will produce little or no $-a$ in compasses placed as in the Defence, and a much smaller $-e$.

These differences do not show themselves in the value of D, which is in fact less in the Resistance than in the Defence, notwithstanding the much more powerful action of the forces which cause it. In order to see them, we must obtain separately the two parts of the quadrantal deviation D, or the value of a and e. This is done in the following Table:—

		Warrior.	Black Prince.	Achilles.	Defence.	Resistance.	Hector.	Valiant.	Royal Oak (wood-built).
Standard ..	From fore-and-aft induction...	+ 0° 6′	− 4° 4′	− 2° 45′	− 2° 42′	− 5° 55′	− 3° 51′	− 2° 14′	− 1° 19′
	From transverse induction ...	+ 8 24	+11 42	+ 9 40	+ 9 44	+12 21	+ 9 15	+ 7 11	+ 4 32
Starboard Steering ...	From fore-and-aft induction...	+ 0 14	− 3 47	− 3 47	− 2 17	− 7 53	− 3 23	− 2 59	− 2 7
	From transverse induction ...	+11 46	+14 28	+12 43	+12 35	+16 33	+11 49	+ 9 54	+ 3 51
Main Deck	From fore-and-aft induction...	......	− 2 35	− 3 9	− 2 10	− 1 02	− 5 58	− 6 56	− 3 51
	From transverse induction ...	......	+15 58	+15 36	+16 58	+15 11	+15 54	+15 14	+ 5 20
Standard ..	a	+·002	−·112	−·079	−·078	−·158	−·109	−·068	−·043
	e	−·256	−·322	−·277	−·278	−·326	−·263	−·214	−·143
Starboard Steering ...	a	+·006	−·100	−·103	−·064	−·193	−·093	−·085	−·066
	e	−·340	−·380	−·343	−·348	−·401	−·325	−·281	−·122
Main Deck	a	......	−·068	−·083	−·048	−·027	−·151	−·176	−·116
	e	......	−·418	−·407	−·434	−·409	−·397	−·380	−·160

The conclusions we have drawn will be seen to be supported by this separation. Thus we see that the Warrior is the only vessel which has a $+a$ and a $+\mathrm{D}$ from fore-and-aft iron. In the Hector and Valiant the D is comparatively small, because the $-a$ is large, the $-e$ small.

In the Resistance the two parts, the difference of which makes up the D, are very much larger than in the Defence, though the resulting value of D is less.

The comparison of the values of D and of a and e in the compasses of the Royal Oak with those in the compasses of the Hector and Valiant is very instructive. These ships are nearly alike in dimension, in the arrangement of the iron-plating, and the position of the compasses. The Royal Oak has an iron upper deck, but is otherwise wood-built. The Hector and Valiant are entirely iron-built.

A first inspection of the Table might lead us to infer that the large value of D in the iron-plated ships is due to the armour-plating at the sides, but the comparison with the Royal Oak shows this not to be the case. In fact a little consideration will show that, as regards longitudinal induction, the effect of armour-plating continued from end to end is to produce a $-a$; that, as regards transverse induction, the effect of the parts which run fore and aft is to produce a small $+e$, and the effect of the transverse parts near the extremities of the ship to produce a small $-e$, so that on the whole the tendency is probably rather to diminish than to increase D. The large value of D in the iron ships is evidently attributable to the increased amount of transverse iron in decks, bulkheads, iron beams, and the iron bottom of the ship, the magnetism of which is, as it were, conducted upwards by the iron sides.

λ.

The value of λ is so closely connected with that of D that it is desirable to consider them together. In the earlier built iron vessels λ was very nearly equal to 1. In the Rainbow, at four stations distributed along nearly the whole length of the ship, λ ranged from ·972 to 1·003. In the Ironsides, the first iron-built sailing ship, it was ·917 at

the steering-compass. In several iron-built ships purchased into the Royal Navy from ten to fifteen years after Mr. AIRY's observations, λ averages at present about ·930. In the iron-plated ships of the present day it ranges from ·700 to ·900.

The following are its values in the iron-plated ships before mentioned.

	Warrior.	Black Prince.	Achilles.	Defence.	Resistance.	Hector.	Valiant.	Royal Oak (wood-built).
Standard compass	·873	·783	·822	·822	·758	·814	·859	·907
Starboard steering......	·833	·760	·777	·794	·703	·791	·817	·906
Main deck	......	·757	·755	·759	·782	·726	·722	·862

The large value in the Warrior is evidently owing to the rifle tower, the small value in the Resistance, as compared to the value in the Defence, to the position of the compasses with respect to the armour bulkheads as above described, and with reference to the armour-plating generally.

Familiarity with the values of $\mathfrak{D}$ and λ in vessels of different classes, is of great importance in enabling us to deduce $\mathfrak{B}$ and $\mathfrak{C}$ by observations made without swinging.

The mathematical theory from which the values of $\mathfrak{D}$ and λ are derived, supposes that the transient induced magnetism to which $\mathfrak{D}$ and $1-\lambda$ owe their values, is instantaneously developed, and as instantaneously destroyed or altered as the ship assumes a new position. This we cannot suppose to be exactly true; but whether the time required for the soft iron to receive its new magnetic state as the ship swings is appreciable has been a matter of doubt. The opinion of the authors of the Report of the Liverpool Compass Committee (an opinion entitled to the greatest weight) was, that an appreciable time was required, and that the value of D in particular might be different according as the vessel was swung slowly or quickly; we have not, however, been able to detect any difference in the values of D which can be attributed to any cause of this nature.

The most remarkable feature, however, in λ and $\mathfrak{D}$ is the change which takes place with the lapse of time, indicating apparently a change in the molecular structure of the soft iron by which it becomes less susceptible of induced magnetism. This is shown clearly in the following Table:—

			Standard.		Starboard steering.		Main deck.	
			λ	$\mathfrak{D}$	λ	$\mathfrak{D}$	λ	$\mathfrak{D}$
Achilles	October	1864	·822	+·121	·777	+·154	·755	+·214
	December	1864	·854	+·116	·819	+·137	·804	+·188
Black Prince.	November	1861	·716	+·145				
	September	1862	·783	+·134	·760	+·184		
	April	1864	·846	+·137				
	November	1864	·849	+·122	·881	+·144		
Defence	February	1862	·822	+·122	·794	+·179	·759	+·254
	December	1863	·853	+·122	·842	+·180	·810	+·230
	April	1864	·857	+·112	·853	+·159	·828	+·233
	October	1864	·852	+·112	·830		·842	+·230
Resistance ...	August	1862	·758	+·111			·782	+·244
	December	1863	·850	+·122			·880	+·219
Royal Oak ...	March	1863	·861	+·047				
	April	1863	·907	+·061	·887	+·067		
	June	1863	·907	+·055	·906	+·031		
Dromedary...	July	1862	·841	+·104				
	December	1862	·861	+·097				

These changes, and particularly that in the value of λ, seem far too great, far too regular, and far too consistent, to be attributed to any cause except some molecular change in the structure of the iron which, with the lapse of time, renders it less susceptible of induced magnetism. Whether this change is accompanied by any change which can affect the strength, the liability to oxidation, or any other qualities of the iron, is a point on which we are not able to offer any information, but we beg to suggest it as a question deserving a careful experimental investigation.

Heeling Error.

As the heeling coefficient depends partly on vertical induction in transverse iron, partly on the mean vertical force arising from permanent magnetism and vertical induction in vertical iron, and as the two conspire when the vertical force of the ship acts downwards, or when μ is greater than unity, and counteract each other when the vertical force acts upwards, or when μ is less than unity, we may expect great differences in the heeling coefficient in different ships. In those which have been built head North, we may expect a large heeling error in compasses near the stern, and a smaller one in compasses near the bow, and the converse in ships built head South. This we find to be the case.

In these cases the uniformity of the heeling coefficients from transverse iron is remarkable, and they are, as might be expected, all of the same sign; the differences, it will be seen, are nearly all in the part which arises from vertical force; this varies from 1° 6′ in the Warrior to $-1°$ 9′ in the Enterprise.

It will be seen that in the wood-built iron-plated ships the vertical force is generally

diminished. This is doubtless the effect of the iron plating, which acts as a $-k$. No doubt in iron-plated iron-built ships the effect is the same, and the heeling error is probably diminished and not increased by the effect of the iron plating. Observations of vertical force have not been made in the main-deck compasses of these ships; but probably there the heeling error would be small, and possibly be a heeling error to leeward.

We must observe that there has not been an opportunity of making an exact comparison of the values of the heeling coefficient deduced from theory with those deduced from actually heeling and swinging the ship. The great amount of labour and time required to heel a ship of the class we are discussing, and swing her, has prevented such observations being made in more than a very small number of cases. In the case of the Warrior, Black Prince, and Defence, advantage was taken of their being heeled at

Class of Ship.	Date.	Name of Ship.	Direction of Head in building.	μ	g	Heeling coefficient from vertical induction in transverse iron.	Heeling coefficient from vertical force and induction in vertical iron.	Heeling coefficient to windward.
Iron ships, iron-plated.	July 1862.	WARRIOR	N. 3° E.	1·399	+·069	+0° 43′	+1° 06′	+1° 49′
	Jan. 1863.	„ *Lisbon*	„	—	+·106	+0 32	+0 50	+1 22
	Sept. 1862.	BLACK PRINCE	S. 20° E.	·945	+·118	+1 01	−0 11	+0 50
	Jan. 1863.	„ *Lisbon*	„	—	+·262	+0 43	+0 09	+0 52
	April 1864.	„	„	·971	+·111	+0 48	−0 05	+0 43
	Oct. 1864.	ACHILLES (Standard aft)	S. 51° 40′ E.	·870	+·194	+0 50	−0 23	+0 27
	Dec. 1864.	„	„	·896	+·210	+0 43	−0 18	+0 25
	Oct. 1864.	„ (Standard forward)	„	1·217	−·172	+0 49	+0 40	+1 29
	Dec. 1864.	„	„	1·240	−·165	+0 37	+0 41	+1 18
	Feb. 1862.	DEFENCE	S. 47° W.	1·040	+·138	+0 51	+0 08	+0 59
	Jan. 1863.	„ *Lisbon*	„	—	+·117	+0 33	−0 03	+0 30
	April 1864.	„	„	·968	+·157	+0 42	−0 06	+0 36
	Aug. 1862.	RESISTANCE	S. 86½° W.	1·071	+·176	+1 04	+0 14	+1 18
	Dec. 1863.	„	„	1·044	+·190	+0 45	+0 08	+0 53
	Feb. 1864.	HECTOR	S. 20° E.	·983	....	+0 48	−0 03	+0 45
	Jan. 1865.	VALIANT	S. 87° W.	1·061	+·120	+0 37	+0 11	+0 48
Wood ships, iron-plated.	April 1863.	ROYAL OAK	Plated S. 49° E.	·896	+·045	+0 24	−0 17	+0 07
	June 1863.	„	„	·882	+·127	+0 23	−0 19	+0 04
	Feb. 1864.	PRINCE CONSORT	Plated S. 39° W.	·848	+·038	+0 16	−0 24	−0 08
	Aug. 1864.	OCEAN	Plated S. 79° E.	·929	+·112	+0 19	−0 34	−0 15
	June 1864.	ENTERPRISE (Iron topsides)	Built and plated S. 56° W.	·622	+·152	+0 37	−1 09	−0 29
Iron ships.	July 1863.	ORONTES	N. 66° W.	1·164	+·056	+0 36	+0 28	+1 04
	Nov. 1863.	TAMAR	West	1·117	+·147	+0 31	+0 20	+0 51
		„ (Binnacle over rudder)	„	1·248	+·294	+0 28	+0 42	+1 10
	Sept. 1863.	WYE	Probably to E.S.E.	1·195	+·252	+0 27	+0 34	+1 0
	Feb. 1863.	CARADOC	Probably to N. by W.	1·002	....	+0 14	+0 01	+0 15
	Feb. 1863.	CLYDE	Probably to N.E.	1·275	....	+0 35	+0 47	+1 22
	Mar. 1863.	INDUSTRY	Probably to S. by E.	·859	....	+0 18	−0 23	−0 05
	June 1863.	CITY OF SYDNEY	Probably to W.N.W.	1·246	....	+0 46	+0 45	+1 31

Lisbon for the purpose of cleaning the bottoms, to swing them at the same time, and the heeling coefficients so obtained correspond very satisfactorily with those obtained in England from observations of horizontal and vertical force. But, unfortunately, at present we have no instances in which the horizontal and vertical forces were observed at the time and place at which the ship was heeled and swung; and it seems very desirable that the theory should be put to the practical test, though there seems no reason to doubt that the results of the two methods would agree within the limits of errors of observation.

g.

g is one of those quantities which it is of importance to be able to estimate with some approach to accuracy, in order that the value of the mean vertical force, or μ, may be determined by observations of the vertical force made with the ship's head on one point only.

The Tables show that this may be done; g, as might be expected, is larger the nearer the stern the Standard Compass is placed, and is negative in compasses placed near the bow.

Achilles	$+\cdot194$
Resistance	$+\cdot176$
Defence	$+\cdot157$
Black Prince	$+\cdot118$
Warrior	$+\cdot069$
Achilles (Standard forward)	$-\cdot172$

There are indications of changes in the value of the heeling coefficient and in the value of g from the lapse of time, corresponding to the changes in the values of $\mathfrak{D}$ and λ; but more extended observations are necessary to show the amount and law of these changes.

To afford a clear view of the general structure of the armour-plated ships, and the position of the several compasses, profile sketches of these ships are given (Plate XI.), and it may be deemed of sufficient interest to add a brief description of their general arrangements as affecting their magnetic characteristics.

The Warrior, Black Prince, and Achilles, of 6100 tons, are types of the largest size iron-built and iron-plated ships of war; they are 380 feet long, 58 feet beam, 26 feet draught of water, propelled by engines of 1250 horse-power, and carry from forty to twenty heavy guns. 3750 tons of iron is used in the construction of the hull, which varies in thickness from $1\frac{1}{4}$ inch near the keel to $\frac{5}{8}$ inch behind the armour-plates. For the Achilles 1200 tons of iron $4\frac{1}{2}$ inches thick was employed for the armour-plating.

The Hector and Valiant of 4100 tons, and the Defence and Resistance of 3700 tons, are types of the medium and smaller-sized iron-built and iron-plated ships of war. In the general features of construction they are similar to the Warrior, Black Prince, and Achilles; all are frigate-built, or with a main deck for the principal battery of guns,

and the only wood used in the hulls, with the exception of teak-wood backing to the armour plates, is for the surface covering of the iron decks, and for the personal arrangements and accommodation of the crews.

In the Warrior, Black Prince, Defence, and Resistance, the armour-plating of $4\frac{1}{2}$-inch iron is not continued to the bow or stern, but where it terminates is continued from side to side of the ship as an armour bulkhead. In the Achilles, Hector, and Valiant, the armour plating is continued round the ship, but of smaller dimensions near the bow and stern, and with corresponding smaller transverse-armour bulkheads.

The Royal Oak, Prince Consort, Caledonia, and Ocean, of 4050 tons, 800 to 1000 horse-power engines, and carrying thirty-five heavy guns, are types of the largest-sized wood-built iron plated-ships; the hull, with the exception of the iron upper deck and its supporting iron beams and uprights, is entirely constructed of wood; the exterior of the hull to 4 feet below the water-line (in this respect similar to the iron-built ships) is plated with $4\frac{1}{2}$-inch iron entirely round.

The Enterprise, of 993 tons, is the type of the smaller-sized wood-built ship; she is constructed to carry four heavy guns within a square battery of $4\frac{1}{2}$-inch iron, and has a continuous armour belt of $4\frac{1}{2}$-inch iron round the ship; the upper deck, deck beams, and top sides are of thin plate-iron.

The Royal Sovereign, of 3765 tons, is an experimental class of vessel; she was originally a wood-built three-decked ship of 110 guns, but now cut down to the lower-gun deck, plated continuously round with $5\frac{1}{2}$-inch iron, and with an iron upper deck and bulworks. The armament of five guns of large calibre is worked within four turrets; the iron frame of these turrets varies in thickness from $5\frac{1}{2}$ to 10 inches; and the largest, arranged to carry two guns, weighs 146 tons.

The internal arrangements of all these classes of ships allow little room for selection in the position of the compasses. The accurate drawings, kindly furnished by the Department of the Controller of the Navy, enables their several positions to be shown with reference to the most important masses of iron.

TABLES OF COEFFICIENTS.

I. Iron-plated, Iron-built Ships.

II. Iron-plated, Wood-built Ships.

III. Iron-built Ships, Her Majesty's Navy.

IV. Iron-built Ships, Mercantile Marine.

Table of Terrestrial Magnetic Elements employed in discussion of magnetic coefficients.

Table I.—Iron-plated, Iron-built Ships.

Ship.	Compass.	Place.	Date.	Approximate coefficients. A	B	C	D	E	Exact coefficients. 𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
Warrior.	Standard.	Greenhithe	...Sept. 16, 17, 1861	+1 7	−24 15	− 7 42	+ 9 23	+0 39	+·019	−·449	−·124	+·164	+·010
(6109 tons),		Portsmouth	...Oct. 15, 17, 1861	−1 0	−22 12	− 5 52	+ 8 56	+0 44	−·017	−·409	−·092	+·155	+·013
Iron-plated,		*Gibraltar*	...*Feb.* 1862	...	−15 51	− 6 0	+ 8 20	+0 23	...	−·293	−·095	+·145	+·006
iron hull,		Portsmouth	...July 28, 29, 1862	−0 12	−17 24	− 7 9	+ 8 27	+1 8	−·003	−·321	−·114	+·148	+·020
40 guns,		*Gibraltar*	...*Nov.* 1862	+1 0	−14 39	− 4 50	+ 8 25	−0 43	+·017	−·272	−·077	+·146	−·012
1250 horse-power.		*Lisbon*	*Heeled* 7½° *to Port*	+0 50	−14 43	+ 6 45	+ 8 9	+0 59	+·015	−·272	+·108	+·143	+·017
Built at Blackwall,		*Lisbon*	*Upright, Jan.* 1863 ...	+0 50	−14 33	− 3 34	+ 7 48	−0 24	+·015	−·269	−·057	+·136	−·007
River Thames; head N. 3° E. magnetic.		*Lisbon*	*Heeled* 7½° *to Starboard*	+0 44	−15 36	−13 37	+ 8 17	−0 45	+·013	−·287	−·216	+·145	−·013
		Devonport	...May 1, 1863	−0 12	−17 10	− 8 18	+ 8 26	−0 32	−·003	−·317	−·132	+·146	−·009
Launched		*Madeira*	......*Dec.* 28, 30, 1863	−1 59	−12 56	− 2 48	+ 7 15	−0 4	−·035	−·239	−·046	+·126	−·001
Dec. 29, 1860.		Plymouth	...June 1864	+0 25	−16 45	− 3 24	+ 8 44	−0 19	+·007	−·311	−·054	+·152	−·005
Plated with head		Portland	......Oct. 28, 1864	−0 17	−16 35	− 4 33	+ 8 45	−0 41	−·005	−·307	−·072	+·152	−·012
generally to N.W.	Starboard steering.	Greenhithe	...Sept. 16, 17, 1861	+0 20	−20 19	− 7 35	+15 28	−0 7	+·006	−·395	−·111	+·268	−·002
		Portsmouth	...Oct. 15, 17, 1861	+0 12	−20 37	− 6 37	+15 51	−0 11	+·003	−·402	−·098	+·273	−·003
		Portsmouth	...July 28, 1862......	−1 48	−15 31	− 7 50	+11 56	+0 43	−·031	−·296	−·121	+·208	+·012
		Devonport	...May 1, 1863	−0 7	−16 28	−10 24	+12 3	−0 31	−·002	−·312	−·160	+·210	−·009
	Main deck.	Greenhithe	...Sept. 11, 1861......	−0 30	−25 56	−12 6	+10 58	−1 15					
		Greenhithe	...Sept. 16, 17, 1861	+0 55	−22 34	− 7 49	+11 43	−1 46					
Black Prince.	Standard.	Greenock[1]	...Nov. 1861	+0 10	+23 0	+ 3 41	+ 8 19	+0 25	+·003	+·422	+·058	+·145	+·007
(6109 tons),		Portsmouth	...Sept. 2, 1862	+0 49	+20 59	+ 4 40	+ 7 38	0 0	+·014	+·383	+·074	+·134	·000
Iron-plated,		*Lisbon*	*Heeled* 6½ *to Port*	+0 57	+15 31	+ 9 11	+ 6 45	+1 50	+·016	+·282	+·148	+·117	+·032
iron hull,		*Lisbon*	*Upright, Jan.* 1863 ...	−0 1	+15 39	+ 3 12	+ 7 24	−1 20	·000	+·288	+·052	+·129	−·023
41 guns, 1250 horse-power.		*Lisbon*	*Heeled* 6½° *to Starboard*	+0 1	+15 14	− 2 6	+ 7 14	−1 25	·000	+·280	−·034	+·126	−·025
Built at Glasgow;		Portland	......June and July 1863	+0 2	+21 8	+ 4 10	+ 7 6	+0 51	·000	+·384	+·067	+·124	+·015
head S. 20° E. magnetic.		*Madeira*	......*Jan.* 1864	−0 25	+13 12	+ 4 29	+ 7 19	−0 6	−·007	+·243	+·072	+·128	−·002
		Lisbon	.........*Jan. and Feb.* 1864	+0 2	+15 8	+ 3 59	+ 7 20	−0 38	·000	+·278	+·064	+·128	−·011
Launched		Portland	......Mar. and Apr. 1864	+1 22	+21 16	+ 5 24	+ 7 54	−0 24	+·024	+·389	+·086	+·137	−·007
Feb. 27, 1861.		Portland	......Oct. 1864	+0 30	+19 5	+ 3 5	+ 7 2	−0 3	+·009	+·349	+·050	+·122	−·001
Plated head South.	Starboard steering.	Portsmouth	...Sept. 2, 1862	+2 59	+20 9	+ 8 10	+10 32	+1 37	+·052	+·379	+·136	+·184	+·028
		Plymouth	...Nov. 1864	+2 19	+19 40	+ 6 13	+ 8 18	+1 45	+·040	+·363	+·103	+·144	+·030
	Main deck.	Portsmouth	...Sept. 2, 1863	−1 9	+27 25	+ 8 4	+13 16	+0 2	−·020	+·516	+·120	+·231	·000

[1] λ observed at Greenock $=·804$, multiplied by earth's horizontal force $·89=·716$.

TABLE I.—Iron-plated, Iron-built Ships.

Maximum of semicircular deviation $\sqrt{B^2+C^2}$ — Amount.	Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$*. — Amount.	Direction.	Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction. Fore-and-aft, a †	Coefficients of horizontal induction. Transverse e †	Part of D from Fore-and-aft induction.	Part of D from Transverse induction.	Mean Vertical force, μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from Vertical induction in transverse iron.	Heeling coefficients from Vertical force and induction in vertical iron.	$\frac{g}{\tan\theta}$ ‡	g †
°		°					° ′	° ′		° ′	° ′	° ′		
25½	·466	195½												
23	·419	192½												
16	·308 ·409	198												
19	·341	199½	·873	1·145	+·002	−·256	+0 6	+ 8 24	1·399	+1 49	+0 43	+1 06	+·028	+·069
15½	·282 ·375	196												
......	·293	158¼												
15	·275 ·345	192	......	......	......	......	......	......	......	+1 22	+0 32	+0 50		
......	·360	217												
19	·344	203	·860	1·163	−·015	−·265	−0 28	+ 8 52						
12½	·243 ·328	191												
17	·314	190												
17¼	·316	193												
21¾	·410	195½												
21¾	·414	193½												
17½	·320	202	·833	1·201	+·006	−·340	+0 14	+11 46						
19½	·352	207	·878	1·139	+·062	−·306	+2 0	+10 04						
23¼	·426	8	·804 ·716	1·396	−·180	−·388	−7 15	+15 40						
20½	·390	11	·783	1·277	−·112	−·322	−4 4	+11 42	·945	+0 50	+1 1	−0 11	+·048	+·118
......	·318	27½												
16	·293 ·369	10½ ...	......	......	......	......	......	......	......	+0 52	+0 43	+ 0 9		
......	·282	353												
20½	·390	10												
14	·254 ·343	16½												
15¾	·286 ·360	13	·778	1·285	−·122	−·322	−4 28	+11 53						
22	·399	12½	·846	1·182	−·038	−·270	−1 19	+ 9 11	·971	+0 43	+0 48	−0 5	+·045	+·111
19⅓	·354	8	·849	1·178	−·047	−·255	−1 36	+ 8 38						
21¾	·404	20	·760	1·316	−·100	−·380	−3 47	+14 28						
20½	·377	16	·881	1·135	+·008	−·246	+0 14	+ 8 4						
28½	·530	13	·757	1·321	−·068	−·418	−2 35	+15 58						

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

TABLE I. (continued).—Iron-plated, Iron-built Ships.

Ship.	Compass.	Place.	Date.	Approximate coefficients. A	B	C	D	E	Exact coefficients. $\mathfrak{A}$	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{D}$	$\mathfrak{E}$
				° ′	° ′	° ′	° ′	° ′					
ACHILLES*. (6121 tons). Iron-cased, iron hull, 20 guns, 1250 horse-power. Built at Chatham, and fully plated in dock; head S. 51° 40′ E. magnetic. Floated out of dock Dec. 24, 1863.	Standard (aft).	Sheerness	...Oct. 12, 13, 1864	−0 16	+19 54	+ 2 56	+ 6 58	−0 56	−·005	+·362	+·047	+·121	−·016
		Plymouth†	...Dec. 5, 1864	−0 35	+19 54	+ 7 38	+ 6 41	−0 32	−·010	+·361	+·123	+·116	−·009
	Standard (forward).	Sheerness	...Oct. 12, 13, 1864	−0 10	+21 42	+ 1 11	+ 7 19	−0 31	−·003	+·396	+·019	+·128	−·009
		Plymouth	...Dec. 5, 1864	+0 39	+19 51	+ 6 15	+ 5 44	−1 01	+·011	+·357	+·102	+·100	−·018
	Starboard steering.	Sheerness	...Oct. 12, 13, 1864	+0 07	+23 31	+ 4 10	+ 8 51	−1 20	+·002	+·432	+·061	+·154	−·023
		Plymouth	...Dec. 5, 1864	−0 55	+23 30	+10 04	+ 7 51	−0 30	−·016	+·427	+·160	+·137	−·009
	Main deck (starboard).	Sheerness	...Oct. 12, 13, 1864	−0 47	+12 42	+ 2 19	+12 13	+0 21	−·014	+·244	+·031	+·214	+·006
		Plymouth	...Dec. 5, 1864	−1 11	+14 17	+ 3 49	+10 46	+1 23	−·021	+·271	+·059	+·188	+·024
DEFENCE. (3720 tons). Iron-plated, iron hull, 16 guns, 600 horse-power. Built on River Tyne; head S. 47° W. magnetic. Launched Apr. 24, 1861. Plated with head S. 19° E. magnetic.	Standard.	Sheerness	...Feb. 17, 18, 1862	−0 28	+25 43	+ 0 17	+ 7 0	+0 5	−·008	+·464	+·005	+·122	+·001
		Baltic Sea	...*July and Aug.* 1862	−0 17	+25 35	− 0 25	+ 6 25	−0 41	−·005	+·463	−·007	+·112	−·012
		Gibraltar	...*Nov.* 15, 1862......	+0 16	+15 21	− 4 15	+ 6 9	+0 25	+·005	+·280	−·069	+·107	+·007
		Lisbon	*Heeled* 7½° *to Port*	+1 47	+16 39	+ 2 49	+ 7 18	+1 50	+·031	+·305	+·045	+·127	+·032
		Lisbon	*Upright, Jan.* 1863 ...	+1 41	+16 26	− 1 5	+ 7 4	+0 42	+·029	+·302	−·018	+·123	+·012
		Lisbon	*Heeled* 7½° *to Starboard*	+1 38	+16 27	− 4 40	+ 7 0	−0 5	+·028	+·301	−·075	+·122	+·001
		Flushing & Portsmouth	March 3, 21, 1863	+0 5	+20 50	− 2 8	+ 6 50	−0 11	+·001	+·379	−·034	+·119	−·003
		Plymouth	...Dec. 1863	+1 6	+22 18	− 0 57	+ 6 59	−0 7	+·019	+·403	−·016	+·122	−·002
		Teneriffe	......*Jan.* 2, 3, 1864 ...	......	......	......	......	......	...	+·292	−·040	+·114	...
		Gibraltar	...*Jan.* 9, 13, 1864...	−1 0	+15 25	− 1 44	+ 6 30	−0 46	−·017	+·282	−·028	+·113	−·013
		Lisbon	.........*Jan. and Feb.* 1864	+0 40	+16 37	− 1 18	+ 6 22	−0 7	+·012	+·303	−·021	+·111	−·002
		Portland	......Mar. and Apr. 1864	+0 21	+21 37	− 0 24	+ 6 26	−0 24	+·005	+·391	−·007	+·112	−·007
		Portland	......Oct. 1864	−0 23	+20 55	− 2 6	+ 6 23	+0 10	−·007	+·379	−·034	+·112	+·003
	Starboard steering.	Sheerness	...Feb. 17, 18, 1862	+0 16	+36 14	+ 0 56	+10 16	+1 7	+·005	+·653	+·014	+·179	+·019
		Plymouth	...Dec. 1863	+1 4	+31 18	− 1 21	+10 19	+0 36	+·019	+·572	−·020	+·180	+·010
		Portland & Downs ...	Apr. and May 1864	......	......	......	......	......	+·014	+·586	−·030	+·159	+·009
		Devonport	...Nov. 1864	......	......	......	......	......	...	+·546	−·056	+·159	...
	Main deck.	Sheerness	...Feb. 17, 18, 1862	−0 51	+36 23	+ 0 42	+14 35	−0 55	−·015	+·669	+·010	+·254	−·016
		Plymouth	...Dec. 1863	+1 16	+26 44	+ 0 34	+13 10	−0 6	+·022	+·505	+·009	+·230	−·002
		Portland & Downs ...	Apr. and May, 1863	......	......	......	......	......	+·019	+·450	+·004	+·233	−·013
		Devonport	...Nov. 1864	......	......	......	......	......	...	+·486	−·030	+·230	...

			$\mathfrak{B}$	$\mathfrak{C}$
* ACHILLES.	Dec. 23, 1863	In dock at Chatham; by observations of deviation and horizontal force on one point, and employing λ and $\mathfrak{D}$ of Oct. 1864 (no machinery on board, or internal fittings)	=+·464	+·323
	Sept. 26, 1864	Complete in equipment; by observations of deviation and horizontal force on one point. Head moored N. 62° W., same λ and $\mathfrak{D}$ as above	=+·377	+·037
	Oct. 11, 1864.	Same observations, λ and $\mathfrak{D}$ as above. Head moored S. 54° 40′ E.	=+·355	+·062

† After having remained in dry dock 25 days. Head S. 79° E. magnetic.

TABLE I. (continued).—Iron-plated, Iron-built Ships.

Maximum of semicircular deviation $\frac{\sqrt{B^2+C^2}}{\text{Horizontal force of ship}}$ $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$*. Amount.		Direction.	Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction. Fore-and-aft, a †	Transverse e †	Part of D from Fore-and-aft induction.	Transverse induction.	Mean Vertical force, μ ‡	Heeling coefficient to *windward*, $\varkappa$	Heeling coefficients from Vertical induction in transverse iron.	Vertical force and induction in vertical iron.	$\frac{g}{\tan\theta}$ ‡	g †
°		°					° ′	° ′		° ′	° ′	° ′		
20¼	·365	7½	·822	1·216	−·079	−·277	−2 45	+ 9 40	·870	+0 27	+0 50	−0 23	+·079	+·194
21	·381	18½	·854	1·171	−·047	−·245	−1 36	+ 8 17	·896	+0 25	+0 43	−0 18	+·084	+·210
21¾	·397	2½	·831	1·202	−·063	−·275	−2 7	+ 9 30	1·217	+1 29	+0 49	+0 40	−·070	+·172
20¾	·371	16	·872	1·147	−·041	−·215	−1 22	+ 7 7	1·240	+1 18	+0 37	+0 41	−·066	+·165
24	·437	8	·777	1·287	−·103	−·343	−3 47	+12 43						
25½	·458	20¾	·819	1·221	−·069	−·293	−2 24	+10 15						
13	·246	7½	·755	1·325	−·083	−·407	−3 9	+15 36						
14¾	·278	12½	·804	1·244	−·045	−·347	−1 36	+12 28						
25¾	·464	360¼	·822	1·217	−·078	−·278	−2 42	+ 9 44	1·040	+0 59	+0 51	+0 8	+·056	+·138
25½	{·463 ·440	359¾												
16	{·288 ·383	346												
16¾	·317	15½ }												
16½	{·303 ·381	356½ }	......	......	......	......	......	......	......	+0 30	+0 33	−0 3		
17	·311	346 }												
21	·391	355												
22½	·403	358	·853	1·172	−·043	−·251	−1 26	+ 8 28						
......	{·294 ·408	352	·846	1·182	−·058	−·250	−1 57	+ 8 31						
15½	{·283 ·376	354	·853	1·172	−·051	−·243	−1 40	+ 8 13						
16½	{·304 ·382	356	·827	1·209	−·081	−·265	−2 49	+ 9 16						
21½	·392	359	·857	1·167	−·071	−·263	−1 33	+ 8 0	·968	+0 36	+0 42	−0 6	+·064	+·157
21	·381	355	·852	1·174	−·053	−·243	−1 46	+ 8 13						
36½	·654	361¼	·794	1·258	−·064	−·348	−2 17	+12 35						
31½	·572	358	·842	1·118	−·006	−·310	−0 14	+10 36						
......	·586	357	·853	1·172	−·036	−·308	−0 21	+ 9 33						
......	·558	348½	·830											
36½	·669	361	·759	1·318	−·048	−·434	−2 10	+16 58						
26¾	·505	361	·810	1·235	−·004	−·376	−0 8	+13 24						
......	·450	360½	·828	1·208	+·021	−·365	+0 41	+12 42						
......	·487	356½	·842	1·188	+·036	−·352	+1 12	+12 4						

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

Table I. (continued).—Iron-plated, Iron-built Ships.

Ship.	Compass.	Place.	Date.	Approximate coefficients. A	B	C	D	E	Exact coefficients. 𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
Resistance. (3710 tons), Iron-plated, iron hull, 16 guns, 600 horse-power. Built at Millwall, River Thames; head S. $86\frac{1}{2}$° W. magnetic. Launched April 11, 1861. Plated with head generally to Westward.	Standard.	Sheerness......	Aug. 25, 26, 1862...	+0 36	+ 8 9	− 9 41	+ 6 17	+0 8	+·010	+·149	−·158	+·111	+·010
		Lisbon	*Jan.* 1863	+1 54	+ 4 5	− 6 27	+ 6 54	+0 59	+·033	+·075	−·105	+·120	+·017
		Portsmouth...	June 19, 1863......	+0 44	+ 8 21	− 8 24	+ 5 48	−0 54	+·013	+·152	−·138	+·101	−·016
		Portsmouth...	Dec. 1863	+1 1	+ 5 46	− 7 22	+ 7 0	−1 59	+·018	+·106	−·120	+·122	−·034
		Malta	*Jan.* 1864	−0 19	+ 1 36	− 6 13	+ 6 45	−1 20	−·005	+·030	−·102	+·117	−·023
		Malta	*Dec.* 27, 1864	−0 4	+ 2 30	− 6 43	+ 5 58	−1 8	−·001	+·044	−·116	+·104	−·020
	Starboard steering.	Sheerness......	Aug. 25, 26, 1862...	+0 24	+ 7 55	−17 15	+ 8 28	+1 9	+·007	+·147	−·274	+·148	+·020
		Portsmouth...	June 19, 1863......	+1 20	+10 41	−13 23	+ 8 56	−0 51	+·023	+·198	−·212	+·155	−·015
		Portsmouth...	Dec. 1863	+2 10	+ 9 42	−12 25	+ 9 43	−0 49	+·038	+·181	−·196	+·170	−·014
	Main deck.	Sheerness......	Aug. 25, 26, 1862...	−0 18	+ 9 24	−17 9	+14 0	+0 21	−·005	+·181	−·260	+·244	+·006
		Portsmouth...	June 19, 1863......	+2 9	+ 9 6	−13 6	+13 25	+1 12	+·020	+·175	−·200	+·232	+·021
		Portsmouth...	Dec. 1863	+3 5	+ 3 41	−11 11	+12 39	−3 30	+·054	+·070	−·173	+·219	−·061
Hector (1). (4089 tons), Iron-cased, iron hull, 28 guns, 800 h.-p. Built at Glasgow; head S. 20° E. magnetic. Launched Sept. 26, 1862. Plated with head N. 55° W. and S. 49° W.	Standard.	Portsmouth...	Feb. 16, 1864	−0 24	+21 53	+ 4 54	+ 5 24	−0 39	−·007	+·392	+·079	+·094	−·011
	Starboard steering.	Portsmouth...	Feb. 16, 1864	+0 37	+30 36	+10 37	+ 8 24	−0 16	+·011	+·545	+·164	+·147	−·005
	Main deck.	Portsmouth...	Feb. 16, 1864	+0 16	+31 22	+13 50	+ 9 47	−0 50	+·004	+·520	+·239	+·170	−·014
Valiant. (4144 tons), Iron-plated, iron hull, 28 guns, 800 h.-p. Built at Millwall, River Thames; head S. 87° W. Launched Oct. 14, 1863. Plated with head generally to Westward.	Standard.	Sheerness	...Jan. 12, 16, 1865...	+1 2	+ 2 30	−12 44	+ 4 54	−0 43	+·018	+·046	−·211	+·085	−·012
	Starboard steering.	Sheerness	...Jan. 12, 16, 1865...	+2 7	+ 7 35	−20 12	+ 6 52	−0 14	+·037	+·138	−·325	+·120	−·004
	Main deck (Starboard).	Sheerness	...Jan. 12, 16, 1865...	+2 35	+ 5 29	−18 39	+ 8 5	−0 12	+·045	+·101	−·297	+·142	−·003

(1) Hector, June 9, 1863. In basin at Portsmouth, by observations of Deviation and Horizontal force on one point, and employing λ and D of February 1864, B = +·398, C = +·159.

TABLE I. (continued).—Iron-plated, Iron-built Ships.

Maximum of semicircular deviation $\frac{\sqrt{B^2+C^2}}{\text{Horizontal force of ship } \sqrt{\mathfrak{B}^2+\mathfrak{C}^2}}$ *. Amount.		Direction.	Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction. Fore-and-aft, a †	Coefficients of horizontal induction. Transverse e †	Part of D from Fore-and-aft induction.	Part of D from Transverse induction.	Mean Vertical force, μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from Vertical induction in transverse iron.	Heeling coefficients from Vertical force and induction in vertical iron.	$\frac{g}{\tan\theta}$ ‡	g †
°		°					° ′	° ′		° ′	° ′	° ′		
12¼	·218	313	·758	1·319	−·158	−·326	−5 55	+12 21	1·071	+1 18	+1 4	+0 14	+·071	+·176
7¾	{·129 ·162	305½												
11¾	·205	317¾												
9½	·160	311½	·850	1·176	−·046	−·254	−1 33	+8 34	1·044	+0 53	+0 45	+0 8	+·076	+·190
6½	{·107 ·158	285½												
7¼	{·124 ·183	291												
19	·312	298	·703	1·423	−·193	−·401	−7 53	+16 33						
17¼	·290	313												
15¾	·266	313												
19½	·316	305	·782	1·279	−·027	−·409	−1 2	+15 11						
16	·266	311												
11¾	·187	292	·880	1·136	+·073	−·313	+2 25	+10 15						
24½	·400	12½	·814	1·228	−·109	−·263	−3 51	+ 9 15	·983	+0 45	+0 48	−0 3	−·005	−·013
33½	·568	16½	·791	1·264	−·093	−·325	−3 23	+11 49						
34¼	·572	25	·726	1·377	−·151	−·397	−5 58	+15 54						
13	·216	282½	·859	1·164	−·068	−·214	−2 14	+ 7 11	1·061	+0 48	+0 37	+0 11	+·048	+·120
21½	·353	293	·817	1·224	−·085	−·281	−2 59	+ 9 54						
19½	·313	288½	·722	1·385	−·176	−·380	−6 56	+15 14						

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

Table II.—Iron-plated, Wood-built Ships.

Ship.	Compass.	Place.	Date.	Approximate coefficients. A	B	C	D	E	Exact coefficients. 𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
Royal Oak. Iron-cased, wood-built, 4056 tons, 35 guns, 800 horse-power. Iron-plated; head S. 49° E. Floated out of dock March 19, 1863.	Standard.	Chatham	Mar. 19, 1863......	......	......	......	......	......	...	+·253	+·287	+·047	...
		Chatham	Apr. 11, 1863......	......	......	......	......	......	...	+·231	+·197	+·061	...
		Sheerness......	June 2, 1863	−0 39	+13 56	+ 7 26	+ 3 9	+0 1	−·011	+·248	+·128	+·055	·000
		Plymouth ...	Jan. 8, 1864	−0 12	+12 20	+10 9	+ 2 19	+0 20	−·003	+·218	+·172	+·040	+·006
		Malta	*Mar.* 1, 1864	−1 9	+ 8 8	+ 6 1	+ 2 58	−0 48	−·020	+·143	+·108	+·052	−·014
	Starboard steering.	Chatham	Apr. 11, 1863......	......	......	......	......	......	...	+·377	+·379	+·067	...
		Sheerness......	June 2, 1863	+0 15	+24 5	+14 25	+ 1 47	+1 17	+·004	+·414	+·241	+·031	+·022
	Main deck.	Sheerness......	June 2, 1863	−1 54	+32 22	+12 47	+ 1 28	−0 11	−·033	+·546	+·210	+·026	−·003
Prince Consort. Iron-cased, wood-built, 4045 tons, 35 guns, 1000 horse-power. Iron-plated; head S. 39° W.	Standard.	Milford	May 25, 1863......	−0 6	+33 39	−13 41	+ 2 18	−0 4	−·001	+·569	−·222	+·040	−·001
		Plymouth ...	Feb. 9, 1864	−0 28	+25 36	− 3 53	+ 3 6	−0 33	−·008	+·447	−·064	+·054	−·010
Caledonia. Iron-cased, wood-built, 4125 tons, 35 guns. 1000 horse-power. Iron-plated; head S. 26° W.	Standard.	Sheerness......	June 15, 1864......	+0 18	+25 47	− 8 21	+ 2 57	+0 20	+·005	+·448	−·138	+·051	+·006
Ocean. Iron-cased, wood-built, 4047 tons, 35 guns. 1000 horse-power. Iron-plated; head S. 79° E.	Standard.	Devonport ...	Aug. 3, 1864	+0 8	+13 2	+15 23	+ 2 31	−0 4	+·002	+·229	+·259	+·044	−·001
Royal Sovereign. Iron-cased, wood-built, turret ship of 5 guns, 3765 tons, 800 horse-power. Iron-plated; head S. 72° E.	Standard.	Portsmouth...	July 21, 22, 1864	−0 3	+12 38	+13 39	+ 7 41	+0 7	−·001	+·233	+·219	+·134	+·002
	Steering wheel (upper deck).	Portsmouth...	July 21, 22, 1864	−1 8	+23 30	−19 40	+13 3	−9 14	−·022	+·487	−·323	+·238	−·159
	Steering wheel (Cap.'s cabin).	Portsmouth...	July 21, 22, 1864	−0 25	+20 11	+ 4 56	+ 6 20	−5 10	−·007	+·364	+·086	+·110	−·090
	Starbd forward (lower deck).	Portsmouth...	July 21, 22, 1864	−0 37	−13 15	+40 15	+15 43	−4 42	−·004	−·277	+·563	+·272	−·078
	Port, forward (lower deck).	Portsmouth...	July 21, 22, 1864	+6 42	−14 35	− 7 8	+13 23	+4 38	+·117	−·286	−·119	+·233	+·081
	Suspended over fore-turret.	Portsmouth...	July 21, 22, 1864	+1 0	−19 33	+ 9 23	+ 8 9	+ 0 1					
Enterprise [1]. (993 tons), 4 guns, 160 h.-p. screw. Built and plated at Deptford; head S.56°W. Launched February 1864.	Standard.	Greenhithe ...	June 7, 1864	+1 24	+14 42	−18 45	+ 2 34	+0 35	+·025	+·257	−·312	+·045	+·010
Wolverene [2]. (703 tons), 21 guns, 400 h. p. screw. Built at Woolwich; head S.S.W. Launched in 1863.	Standard.	Greenhithe ...	May 31, 1864......	+0 23	+14 10	− 2 11	+ 3 20	+0 46	+·007	+·253	−·036	+·058	+·013

[1] Wood bottom, Iron-cased, with central iron battery. Iron topsides, decks and beams.

[2] Wood hull, iron beams and stanchions.

TABLE II.—Iron-plated, Wood-built Ships.

Maximum of semicircular deviation $\frac{\sqrt{B^2+C^2}}{\text{Horizontal force of ship } \sqrt{\mathfrak{B}^2+\mathfrak{C}^2}}$*.			Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction.		Part of D from		Mean Vertical force. μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from		$\frac{g}{\tan\theta}$ ‡	g †
Amount.		Direction.			Fore-and-aft, a †	Transverse e †	Fore-and-aft induction.	Transverse induction.			Vertical induction in transverse iron.	Vertical force and induction in vertical iron.		
°		°					° ′	° ′		° ′	° ′	° ′		
......	·382	48¾	·861	1·162	−·098	−·178	−3 16	+ 6 2						
......	·304	40½	·907	1·102	−·038	−·148	−1 12	+ 4 39	·896	+0 7	+0 24	−0 17	+·018	+·045
15¾	·280	27½	·907	1·102	−·043	−·143	−1 19	+ 4 32	·882	+0 4	+0 23	−0 19	+·052	+·127
16	·278	38												
10	{·179 ·264}	37												
......	·534	45	·887	1·127	−·054	−·172	−1 43	+ 5 30						
28	·480	30	·906	1·104	−·066	−·122	−2 7	+ 3 51						
34¾	·586	21	·862	1·160	−·116	−·160	−3 51	+ 5 20						
36½	·612	339	·840	1·190	−·126	−·194	−4 18	+ 6 36						
26	·452	352	·950	1·053	+·001	−·101	0 0	+ 3 6	·848	−0 8	+0 16	−0 24	+·015	+·038
27	·469	343	·895	1·117	−·059	−·151	−1 53	+ 4 46						
20⅓	·346	48½	·923	1·083	−·036	−·118	−1 9	+ 3 40	·929	−0 15	+0 19	−0 34	+·045	+·112
18½	·320	43½	·912	1·097	+·044	−·204	+1 5	+ 6 36						
30½	·584	326	·980	1·020	+·202	−·242	+5 58	+ 7 7						
20¾	·374	13½	·917	1·091	+·028	−·184	+0 34	+ 5 45						
42½	·629	116	·783	1·277	−·003	−·431	−0 2	+15 55						
16¼	·310	203	·811	1·233	·000	−·379	0 0	+13 25						
23¾	·406	309½	·817	1·224	−·146	+·220	−5 6	+ 7 44	·622	−0 29	+0 37	−1 9	+·062	+·152
14¼	·256	352	·962	1·039	+·018	−·094	+0 35	+ 2 45	·953	+0 7	+0 14	−0 7		

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

Table III.—Iron-built Ships, Her Majesty's Navy.

Ship.	Compass.	Place.	Date.	Approximate coefficients. A	B	C	D	E	Exact coefficients. $\mathfrak{A}$	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{D}$	$\mathfrak{E}$
				° ′	° ′	° ′	° ′	° ′					
Orontes. (2812 tons), 4 guns, 500 h.-p., screw. Built at Birkenhead; head N. 66° W. magnetic. Launched Nov. 22, 1862.	Standard.	Plymouth	May 26, 1863	−0 31	− 7 45	−12 20	+5 46	−0 24	−·009	−·141	−·203	+·100	−·007
		Portsmouth	July 7, 1863	−0 2	− 6 55	−12 0	+5 30	−0 13	·000	−·125	−·198	+·096	−·004
		C. of Good Hope,	*Nov.* 1864	−1 40	− 9 39	−10 41	+5 49	0 0	−·029	−·177	−·178	+·101	·000
	Starboard steering.	Portsmouth	July 7, 1863	−0 43	−10 27	−13 7	+7 16	−0 22	−·012	−·191	−·213	+·126	−·006
Tamar. (2812 tons), 4 guns, 500 h.-p., screw. Built at Millwall, River Thames; head West. Launched Jan. 5, 1863.	Standard.	Sheerness	Nov. 21, 23, 1863	+0 18	+ 1 42	−10 49	+3 18	+0 33	+·005	+·031	−·184	+·058	+·010
		Portsmouth	Oct. 1864	+0 4	+ 2 11	− 5 26	+3 11	+0 22	+·001	+·038	−·095	+·056	+·006
	Starboard steering.	Sheerness	Nov. 21, 23, 1863	−1 50	+ 7 15	−17 14	+3 27	+0 8	−·032	+·128	−·288	+·060	+·002
Adventure. (1794 tons), 400 horse-power, screw. Built at Birkenhead. Launched Feb. 17, 1855.	Standard.	Greenhithe	April 26, 1862	+0 2	− 4 5	+10 59	+2 56	+0 26	·000	−·073	+·186	+·051	+·007
		Greenhithe	Oct. 28, 1862	+0 8	− 3 59	+10 59	+2 53	+0 10	+·002	−·071	+·186	+·050	+·003
		Yokohama, Japan	*Nov.* 11, 1864	……	− 3 28	+ 8 4	+2 49	−0 19	……	−·061	+·139	+·049	+·005
Dromedary. (647 tons), 100 horse-power, screw.	Standard.	Greenhithe	July 8, 1862	+0 32	+ 5 0	−11 50	+6 0	+0 14	+·009	+·091	−·194	+·104	+·044
		Greenhithe	Dec. 16, 1862	+0 21	+ 4 59	−10 55	+5 33	+0 44	+·006	+·091	−·179	+·097	+·013
Wye. (700 tons), 100 horse-power, screw.	Standard.	Greenhithe	Sept. 1, 1863	+0 25	+ 3 24	+10 50	+1 31	+0 5	+·007	+·059	+·186	+·026	+·001
Caradoc. (676 tons), Paddle-wheel, 350 h.-p. Built at Blackwall. Launched July 1847.	Standard.	Greenhithe	Feb. 12, 1863	−0 43	−13 28	− 2 54	+2 3	−0 7	−·012	−·238	−·049	+·036	−·002
Industry. (638 tons), Screw, 80 horse-power. Built at Blackwall. Launched 1854.	Standard.	Greenhithe	March 14, 1863	−0 13	+11 32	− 2 16	+2 58	−0 6	−·004	+·206	−·038	+·052	−·002
Supply. (638 tons), Screw, 80 horse-power. Built at Blackwall. Launched June 1854.	Standard.	Greenhithe	Oct. 17, 1863	−0 12	−13 32	− 1 40	+2 55	+0 16	−·003	−·240	−·028	+·051	+·004

TABLE III.—Iron-built Ships, Her Majesty's Navy.

Maximum of semicircular deviation $\sqrt{B^2+C^2}$ / Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$*. Amount.		Direction.	Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction. Fore-and-aft, a †	Transverse e †	Part of D from Fore-and-aft induction.	Transverse induction.	Mean Vertical force, μ ‡	Heeling coefficient to windward, $\varkappa$	Heeling coefficients from Vertical induction in transverse iron.	Vertical force and induction in vertical iron.	$\frac{g}{\tan\theta}$ ‡	g †
°		°					° ′	° ′		° ′	° ′	° ′		
14½	·247	235												
14	·234	238	·875	1·143	−·041	−·209	−1 22	+6 40	1·164	+1 4	+0 36	+0 28	+·023	+·056
14½	{·251 ·293	225												
17	·286	228	·862	1·160	−·029	−·247	−0 58	+8 13						
11	·187	279½	·870	1·150	−·080	−·180	−2 38	+5 58	1·117	+0 51	+0 31	+0 20	+·060	+·147
6	·102	292												
18¾	·315	294	·886	1·129	−·061	−·167	−2 0	+5 27	1·248	+1 10	+0 28	+0 42	+·120	+·294
11¾	·200	111	·922	1·085	−·031	−·125	−1 0	+3 56						
11¾	·199	111	·918	1·090	−·035	−·129	−1 8	+4 1						
9	{·151 ·249	·113												
12¾	·215	295	·841	1·186	−·072	−·246	−2 21	+8 21						
12	·201	297	·861	1·161	−·056	−·222	−1 50	+7 28						
11½	·195	72	·869	1·151	−·108	−·154	−3 34	+5 3	1·195	+1 0	+0 27	+0 34	+·103	+·252
13¾	·243	191½	·945	1·058	−·021	−·089	−0 38	+2 42	1·002	+0 15	+0 14	+0 1		
11¾	·209	349½	·937	1·067	−·014	−·112	−0 41	+3 40	·859	−0 5	+0 18	−0 23		
13¾	·242	186½	·925	1·081	−·028	−·122	−0 55	+3 47						

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

TABLE IV.—Iron-built Ships, Mercantile Marine.

Ship.	Compass.	Place.	Date	Approximate coefficients. A	B	C	D	E	Exact coefficients. 𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
RAINBOW[1]	Station No. 1	Deptford......	July and Aug. 1838	+0 40	−50 36	−11 4	+1 23	+0 38	+·012	−·802	−·173	+·024	+·011
	„ No. 2	......		+0 35	−18 45	−12 57	+2 30	+0 2	+·010	−·327	−·217	+·044	+·001
	„ No. 3	......		+0 42	−15 46	−10 39	+3 7	−0 2	+·012	−·279	−·181	+·054	−·001
	„ No. 4	......		+0 5	− 8 5	− 9 33	+3 26	+0 2	+·001	−·145	−·161	+·060	+·001
IRONSIDES[2].........	Binnacle, or steering.	Liverpool ...	Oct. 27, 1838	0 · 0	−24 16	+20 50	+2 15	−0 1	·000	−·416	+·346	+·039	·000
GREAT EASTERN[3]	Standard position.	River Thames...	Sept. 7, 1859 ...	−0 10	+23 13	+25 38	+4 21	−0 37	−·003	+·402	+·408	+·076	−·011
		Portland	Sept. 12, 1859...	−1 3	+22 42	+16 43	+4 44	−0 45	−·018	+·400	+·272	+·082	−·013
	Compass aft on platform.	River Thames...	Sept. 7, 8, 1859	−1 40	+13 34	+22 41	+7 55	−0 12	−·029	+·247	+·359	+·138	−·003
	Compass on fore bridge.	River Thames...	Sept. 8, 1859 ...	+0 3	+31 56	+17 47	+4 31	−0 9	+·001	+·551	+·282	+·079	−·003
CLYDE	Standard position.	Greenhithe ...	Feb. 21, 1863	+0 41	− 7 56	+ 7 25	+4 43	+0 8	+·012	−·143	+·124	+·082	+·002
CITY OF SYDNEY...	Standard position.	Greenhithe ...	June 13, 1863......	+1 27	− 3 29	−18 51	+4 32	+0 23	+·025	−·063	−·311	+·079	+·007

[1] Station No. 1, (near the binnacle) 13 ft. 2 in. distant from the extreme part of stern, 4 ft. $0\frac{1}{2}$ in. from deck.
„ 2, 31 9 „ „ „ „
„ 3, 48 3 „ „ „ „
„ 4, {151 6 „ / 47 0 from knight head of stem} „ „
See Philosophical Transactions, 1839, Part I. p. 167.

[2] See Philosophical Transactions, 1839, Part I. p. 206.

[3] See Philosophical Transactions, 1860, Part II. p. 375.

Table of Terrestrial Magnetic Elements. [1864.]

Place.	In British absolute units. Horizontal force.	Vertical force.	Dip θ.	Tan θ.	Horizontal force at Greenwich being unit*. Horizontal force.	Vertical force.
			° ′			
Greenwich	3·83	+ 9·53	+68 7	+2·49	1·00	+2·49
Greenhithe.........	3·84	+ 9·50	+68 5	+2·48	1·00	+2·48
Sheerness	3·83	+ 9·50	+68 2	+2·48	1·00	+2·48
Portsmouth	3·86	+ 9·48	+67 50	+2·45	1·01	+2·47
Portland............	3·88	+ 9·50	+67 45	+2·44	1·01	+2·47
Plymouth	3·86	+ 9·54	+67 58	+2·47	1·01	+2·49
Milford	3·62	+ 9·80	+69 44	+2·71	·95	+2·56
Greenock	3·38	+10·04	+71 23	+2·97	·88	+2·63

* NOTE.— For British absolute units multiply by 3·83.
For Foreign absolute units multiply by 1·76.

TABLE IV.—Iron-built Ships, Mercantile Marine.

Maximum of semicircular deviation $\frac{\sqrt{B^2+C^2}}{\text{Horizontal force of ship } \sqrt{\mathfrak{B}^2+\mathfrak{C}^2}}$*. Amount.		Direction.	Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction. Fore-and-aft a †	Transverse e †	Part of D from Fore-and-aft induction.	Part of D from Transverse induction.	Mean vertical force, μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from Vertical induction in transverse iron	Heeling coefficients from Vertical force and induction in vertical iron.	$\frac{g}{\tan\theta}$ ‡	g †
°		°					° ′	° ′		° ′	° ′	° ′		
52	·822	192	·984	1·016	+·008	−·040	+0 14	+1 9						
22¾	·392	213	·972	1·029	+·015	−·071	+0 24	+2 7						
19	·332	213	1·003	·997	+·057	−·050	+1 40	+1 26						
12½	·217	228	·999	1·001	+·060	−·060	+1 43	+1 43						
32	·542	140	·914	1·094	−·050	−·122	−1 33	+3 50						
34½	·574	45½	·791	1·264	−·072	−·192	−3 50	+8 13						
28	·484	34½	·775	1·291	−·082	−·209	−4 4	+8 48						
26½	·438	55½	·897	1·115	+·066	−·182	+0 38	+7 18						
36½	·619	27	·892	1·121	−·038	−·178	−4 38	+9 16						
11	·189	139	·870	1·149	−·059	−·201	−1 57	+6 39	1·275	1 22	+0 35	+0 47		
19½	·158	258½	·816	1·225	−·120	−·248	−4 8	+8 44	1·246	1 31	+0 46	+0 45		

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

Table of Terrestrial Magnetic Elements. [1864.]

Place.	In British absolute units. Horizontal force.	In British absolute units. Vertical force.	Dip θ.	Tan θ.	Horizontal force at Greenwich being unit ‖. Horizontal force.	Horizontal force at Greenwich being unit ‖. Vertical force.
Lisbon	4·82	+8·46	+60° 23′	+1·76	1·26	+2·21
Gibraltar	5·09	+7·89	+57 9	+1·55	1·33	+2·06
Madeira	5·17	+8·27	+57 55	+1·60	1·35	+2·16
Teneriffe	5·44	+8·10	+56 10	+1·49	1·42	+2·12
Malta	5·65	+7·29	+52 20	+1·29	1·47	+1·90
Simons Bay, Cape of Good Hope	4·48	−6·43	−55 8	−1·44	1·17	−1·68
Yokohama, Japan	6·32	+7·08	+48 10	+1·12	1·65	+1·85

§ NOTE.— For British absolute units multiply by 3·83. For Foreign absolute units multiply by 1·76.

ON THE EFFECT ON THE COMPASS OF PARTICULAR MASSES OF SOFT IRON IN A SHIP*.

The form of the general equations for the effect of the soft iron of a ship on the compass does not, as we have seen, depend on the form, position, or inductive capacity of the iron. They involve, it is true, nine coefficients which depend on these particulars, but the data of the problem are in general not these particulars, but the effects which they cause in certain definite positions of the ship. This is fortunate, because, while the form of the general equations is obtained at once from very simple physical considerations, and while the special formulæ required are deduced from these by simple trigonometrical operations, and the coefficients are then deduced from the observations by a simple arithmetical operation, the *à priori* determination of the effect on the compass of given masses of iron is, in all but the very simplest cases, a matter of great and generally insuperable difficulty.

It is however in all cases interesting, and in some cases important, to be able to form an approximate estimate of the nature and amount of the effects on the compass of particular masses of iron, and although the precise cases of masses of iron in which the problem admits of an exact solution may not often occur, yet cases frequently occur of masses of iron sufficiently resembling them to have much light thrown on their effects by the knowledge of the effect of the simpler bodies which they most nearly resemble.

The most general case for which the problem can be solved is that of ellipsoids and ellipsoidal shells, including the forms into which these degenerate, as spheres, spheroids, plates, cylinders, &c., but the general solution is so extremely unmanageable, in its practical application, that it is more convenient to consider the simpler cases independently. The cases which we shall consider are—

1. Infinitely thin rods of finite or infinitesimal length.
2. Infinitely thin plates of finite dimensions magnetized longitudinally.
3. Infinite plates of finite thickness magnetized perpendicularly.
4. Spheres.
5. Spherical shells.
6. Infinitely long cylinders magnetized perpendicularly.
7. Infinitely long cylindrical shells magnetized perpendicularly.

A little consideration will show that there is hardly any arrangement of iron in a ship which does not bear more or less resemblance to one or other of these cases.

The physical theory of COULOMB, on which POISSON'S mathematical theory is based, supposes, as is well known, that there is no separation of two kinds of magnetism except within infinitely small elements of the iron; but on this theory, if the iron be homoge-

* I beg to express my obligations to Professor W. THOMSON for much of what is contained in this part of the paper, and at the same time to express my hope that he may be induced to complete the promised Treatise on the Mathematical Theory of Magnetism, part of which was published in the Phil. Trans. 1851.—A. S.

neous, the result on all external bodies is precisely the same as that of a certain distribution of North and South magnetism on the surface of the iron.

To avoid the ambiguity which arises from the use of the terms "North" and "South" magnetism, we shall speak of the magnetism of the north end of the needle and the southern hemisphere of the earth as *red* magnetism, of the south end of the needle and the northern hemisphere as *blue* magnetism.

I. *An infinitely thin rod.*

Let S be the area of a section of the rod, F the component of the earth's force in the direction of the rod, and $\varkappa$ a coefficient depending on the inductive capacity of the iron.

Each end of the rod will have a quantity of free magnetism $=\varkappa\mathrm{SF}$, the magnetism being red at the north end, blue at the south end of the rod.

If x, y, z be the coordinates, r the distance of the blue end, x', y', z' the coordinates, r' the distance of the red end, l the length of the rod, X, Y, Z the components of the earth's force, then the effect of the rod on a red particle at the origin is a force

$$\text{Towards } x=\varkappa\mathrm{S}\left(\frac{x}{r^3}-\frac{x'}{r'^3}\right)\left\{\frac{x'-x}{l}\mathrm{X}+\frac{y'-y}{l}\mathrm{Y}+\frac{z'-z}{l}\mathrm{Z}\right\},$$

$$\text{Towards } y=\varkappa\mathrm{S}\left(\frac{y}{r^3}-\frac{y'}{r'^3}\right)\left\{\frac{x'-x}{l}\mathrm{X}+\frac{y'-y}{l}\mathrm{Y}+\frac{z'-z}{l}\mathrm{Z}\right\},$$

$$\text{Towards } z=\varkappa\mathrm{S}\left(\frac{z}{r^3}-\frac{z'}{r'^3}\right)\left\{\frac{x'-x}{l}\mathrm{X}+\frac{y'-y}{l}\mathrm{Y}+\frac{z'-z}{l}\mathrm{Z}\right\}.$$

If the rod be infinitely short, and $x'-x=dx$, $y'-y=dy$, $z'-z=dz$, $l=ds$, then force

$$\text{Towards } x=\varkappa\mathrm{S}\frac{ds}{r^3}\left\{3\frac{x}{r}\left(\frac{x}{r}\frac{dx}{ds}+\frac{y}{r}\frac{dy}{ds}+\frac{z}{r}\frac{dz}{ds}\right)-\frac{dx}{ds}\right\}\left\{\frac{dx}{ds}\mathrm{X}+\frac{dy}{ds}\mathrm{Y}+\frac{dz}{ds}\mathrm{Z}\right\},$$

$$\text{Towards } y=\varkappa\mathrm{S}\frac{ds}{r^3}\left\{3\frac{y}{r}\left(\frac{x}{r}\frac{dx}{ds}+\frac{y}{r}\frac{dy}{ds}+\frac{z}{r}\frac{dz}{ds}\right)-\frac{dy}{ds}\right\}\left\{\frac{dx}{ds}\mathrm{X}+\frac{dy}{ds}\mathrm{Y}+\frac{dz}{ds}\mathrm{Z}\right\},$$

$$\text{Towards } z=\varkappa\mathrm{S}\frac{ds}{r^3}\left\{3\frac{z}{r}\left(\frac{x}{r}\frac{dx}{ds}+\frac{y}{r}\frac{dy}{ds}+\frac{z}{r}\frac{dz}{ds}\right)-\frac{dz}{ds}\right\}\left\{\frac{dx}{ds}\mathrm{X}+\frac{dy}{ds}\mathrm{Y}+\frac{dz}{ds}\mathrm{Z}\right\}.$$

If the rod be in the plane of x, y and parallel to the axis of x, then z, dy and $dz=0$, and force

$$\text{Towards } x=\frac{\varkappa\mathrm{S}l}{r^3}\left(3\frac{x^2}{r^2}-1\right)\mathrm{X},$$

$$\text{Towards } y=\frac{\varkappa\mathrm{S}l}{r^3}\,3\frac{xy}{r^2}\mathrm{X},$$

$$\text{Towards } z=0.$$

If the rod be in the axis of x, then $x=r$, and the force is

$$2\frac{\varkappa\mathrm{S}l}{r^3}\mathrm{X} \text{ in the direction of } +x.$$

If the rod be in the axis of y, then $x=0$, and the force is

$$\frac{\varkappa\mathrm{S}l}{r^3}\mathrm{X} \text{ in the direction of } -x.$$

The product $\varkappa\mathrm{S}l\mathrm{X}$ is called the *moment* of the magnetic rod.

We will now pause to state what is known of the value of $\varkappa$ for iron of different kinds.

The coefficient $\varkappa$ is the quantity so designated by NEUMANN in Crelle's Journal, vol. xxxvii. p. 21, WEBER in Götting. Trans. vol. vi. p. 20, and THALEN in Nov. Act. Soc. Reg. Upsal. 1861.

It is related to the k of POISSON'S papers in the fifth volume of the 'Mémoires de l'Institut,' and to the g of GREEN'S celebrated "Essay on the Mathematical Theories of Electricity and Magnetism" (Nottingham, 1828; reprinted in Crelle's Journal, vol. xlvii.), by the equation

$$k=g=\frac{\frac{4\pi}{3}\varkappa}{1+\frac{4\pi}{3}\varkappa}.$$

GREEN, in the essay referred to, finds, from some experiments of COULOMB on steel wire,

$$g=\cdot986636,$$

whence

$$\varkappa=17\cdot625.$$

WEBER finds the following values of $\varkappa$:

Steel tempered to glass hardness and already magnetized	4·091
Steel tempered to glass hardness with no permanent magnetism	4·934
Soft steel	5·61
Soft iron	36

THALEN finds, from six specimens of soft iron carefully annealed, the following values:

Specimen.	$\varkappa$.
1	34·58
2	27·24
3	45·26
4	32·25
5	44·23
6	36·96
Mean	36·75

From observations of iron bars given by SCORESBY in his 'Magnetical Investigations,' vol. ii. p. 320, we derive

	$\varkappa$.
Iron rod, not struck	16·77
Iron rod, struck	44·07

From observations which we have made with a rod of iron $\frac{7}{16}$ths of an inch in diameter, 3 feet long, we have found

	$\varkappa$.
Iron, not struck	12·48
Iron, struck several sharp blows, about	80

Hence probably in the iron plates used in ship-building $\varkappa$ may vary from 10 to 30.

2. *An infinitely thin plate of finite dimensions magnetized longitudinally.*

If F be the component of the earth's magnetism in the plane, and perpendicular to any part of the edge, we shall have a distribution of red magnetism on the northern edge of the plate, of blue magnetism on the southern; and if m be the thickness of the plate, then the force exerted by a part of the blue edge of length ds, or a red particle at a distance r, will be

$$\varkappa \mathrm{F} \frac{mds}{r^2},$$

and the effect of the whole edge will be given by ordinary integration. Such a plate may in fact be considered as a collection of thin iron rods laid side by side, parallel to the direction of the component of the earth's force which we are considering.

3. *An infinite plate of finite thickness magnetized perpendicularly.*

Let F be the component of the earth's force perpendicular to the plate.

The northern surface of the plate will have a distribution of red free magnetism, the southern surface of blue; the amount of each on an element of surface $=d\mathrm{S}$ being

$$\frac{\varkappa \mathrm{F}}{1+4\pi\varkappa} d\mathrm{S}.$$

Each surface will exercise a force in a direction perpendicular to the plate of $\frac{2\pi\varkappa}{1+4\pi\varkappa}\mathrm{F}$ on a red particle anywhere situate.

Hence the effect of the one surface, in the case of an *external* particle, will be to neutralize the effect of the other.

On an *internal* particle, both surfaces acting in the same direction, the force will be

$$\frac{4\pi\varkappa}{1+4\pi\varkappa}\mathrm{F} \text{ to South.}$$

4. *Sphere.*

The distribution of free magnetism on the surface of a sphere will of course be symmetrical with regard to two poles and an axis parallel to the direction of dip, the free magnetism being red in the northern half of the sphere, blue in the southern; the amount on a unit of surface at either pole will be

$$\mathrm{I}=\frac{\varkappa}{1+\frac{4}{3}\pi\varkappa}\mathrm{F},$$

and at a point at the extremity of a radius making an angle α with the axis

$$\mathrm{I}\cos\alpha=\frac{\varkappa}{1+\frac{4}{3}\pi\varkappa}\,.\,\mathrm{F}\cos\alpha.$$

The effect on a red particle at a distance r from the centre of the sphere, and in a

direction making an angle α with the axis, p being the radius of the sphere, is

$$\frac{\frac{4\pi}{3}\varkappa}{1+\frac{4\pi}{3}\varkappa}\mathrm{F}\frac{p^3}{r^3}(3\cos^2\alpha-1)\ \textit{to North},$$

$$\frac{\frac{4\pi}{3}\varkappa}{1+\frac{4\pi}{3}\varkappa}\mathrm{F}\frac{p^3}{r^3}\,3\sin\alpha\cos\alpha\ \textit{from axis}.$$

Hence, at the pole, *but outside the sphere*, it is

$$\frac{\frac{8\pi}{3}\varkappa}{1+\frac{4\pi}{3}\varkappa}\mathrm{F} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{to North.}$$

At the equator, and anywhere inside the sphere, it is

$$\frac{\frac{4}{3}\pi\varkappa}{1+\frac{4}{3}\pi\varkappa}\mathrm{F} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{to South.}$$

The sphere, therefore, acts on external particles precisely as an infinitely small magnet of moment $\frac{\frac{4}{3}\pi\varkappa}{1+\frac{4}{3}\pi\varkappa}p^3\mathrm{F}$ held in the direction of the lines of force.

Here we may pause to observe the very remarkable fact that while the effect of a thin rod or plate magnetized in a direction parallel to itself is nearly proportional to $\varkappa$, the effect of a plate magnetized at right angles to its plane, or of a solid sphere, is almost independent of the value of $\varkappa$.

Thus, taking WEBER'S observations, the values of $\varkappa$ for steel and soft iron are nearly 5 and 36. A soft iron rod or thin plate magnetized in the direction parallel to itself would therefore have more than seven times the effect of a steel rod or plate of the same dimensions; but in the case of spheres the proportion of the effects would be

$$\frac{5}{5{\cdot}24}:\frac{36}{36{\cdot}24}={\cdot}954:{\cdot}993$$
$$=\ 24 \quad 25 \text{ nearly};$$

or the effect of the hardest steel sphere is within 4 per cent. of the effect of a similar sphere of soft iron, and within 5 per cent. of the effect of a similar sphere of a substance infinitely susceptible of induction, and hammering such a sphere would make no perceptible difference in its effect.

At the equator outside, and anywhere in the interior, the force of the sphere, as we have said, is

$$\frac{\frac{4\pi}{3}\varkappa\mathrm{F}}{1+\frac{4\pi}{3}\varkappa}\ \text{to South};$$

this force therefore would, within 5 per cent. in the case of a steel sphere, and within 1 per cent. in the case of a soft iron sphere, neutralize the effect of the earth's magnetism.

5. *Spherical Shell.*

Let p be the radius of the outer surface, q of the inner.

There will be a distribution of free magnetism on the outside similar to that on the sphere, but in the case of the shell

$$\mathrm{I}=\varkappa\mathrm{F}\frac{1+\frac{8\pi}{3}\varkappa\left(1-\frac{q^3}{p^3}\right)}{1+4\pi\varkappa+\frac{4\pi}{3}\cdot\frac{8\pi}{3}\varkappa^2\left(1-\frac{q^3}{p^3}\right)}.$$

There will be a similar distribution of free magnetism, but of the opposite kind, in the interior surface, such that if I′ represent the amount of *blue* magnetism on a unit of surface at the north pole of the interior surface,

$$\mathrm{I}'=\varkappa\mathrm{F}\frac{1}{1+4\pi\varkappa+\frac{4\pi}{3}\cdot\frac{8\pi}{3}\varkappa^2\left(1-\frac{q^3}{p^3}\right)}.$$

Hence for an external particle the coefficient will be

$$\mathrm{I}-\mathrm{I}'\frac{q^3}{p^3}=\varkappa\mathrm{F}\frac{\left(1+\frac{8\pi}{3}\varkappa\right)\left(1-\frac{q^3}{p^3}\right)}{1+4\pi\varkappa+\frac{4\pi}{3}\cdot\frac{8\pi}{3}\varkappa^2\left(1-\frac{q^3}{p^3}\right)}$$

$$=\frac{\varkappa\mathrm{F}}{1+\frac{4\pi}{3}\varkappa}\cdot\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{3}{8\pi\varkappa}}$$

nearly, if $\varkappa$ be large and $1-\frac{q}{p}$ small.

If $1-\frac{q}{p}$ be infinitely small, the intensity both outside and inside at the North end is $=\frac{\varkappa}{1+4\pi\varkappa}\cdot\mathrm{F}$, or the same as in a plate, as might be expected.

Mr. BARLOW found that in a shell of $\frac{1}{30}$th of an inch thick and 10 inches diameter the effect was $\frac{2}{3}$ that of a solid sphere, whence

$$\frac{\frac{1}{150}}{\frac{1}{150}+\frac{3}{8\pi\varkappa}}=\frac{2}{3},$$

$$\text{or } \varkappa=\frac{112\cdot5}{\pi}$$

$$=35\cdot8,$$

which agrees closely with the previous results.

The coefficient for the force on a point in the interior of the spherical shell is

$$-\frac{4\pi}{3}(\mathrm{I}-\mathrm{I}')$$

$$=-\frac{4\pi}{3}\varkappa\mathrm{F}\frac{\frac{8\pi}{3}\varkappa\left(1-\frac{q^3}{p^3}\right)}{1+4\pi\varkappa+\frac{4\pi}{3}\cdot\frac{8\pi}{3}\cdot\varkappa^2\left(1-\frac{q^3}{p^3}\right)}$$

$$=-\mathrm{F}\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{3}{8\pi\varkappa}}$$

nearly, when $\varkappa$ is large and $1-\frac{q}{p}$ small; and the whole directive force in the interior will in that case be

$$=\frac{\mathrm{F}}{1+\frac{8\pi\varkappa}{3}\left(1-\frac{q}{p}\right)};$$

and therefore if the shell be thick it will be nearly zero, the residual force being inversely as the thickness; if the shell be thin, the loss of force will be nearly proportional to the thickness.

6. *Infinite cylinder, magnetized at right angles to its length.*

Radius $=p$.

The intensity of red magnetism on a point in the surface at an angle α from North is

$$\mathrm{I}=\frac{\varkappa\mathrm{F}\cos\alpha}{2\pi\varkappa+1}.$$

The effect on a red particle at a distance r, r making an angle α with the North and South axis of a perpendicular section, is

$$\frac{2\pi\varkappa}{2\pi\varkappa+1}\mathrm{F}\frac{p^2}{r^2}\cos 2\alpha \quad . \quad . \quad . \quad \text{to North,}$$

$$\frac{2\pi\varkappa}{2\pi\varkappa+1}\mathrm{F}\frac{p^2}{r^2}\sin 2\alpha \quad . \quad . \quad . \quad \text{to East.}$$

7. *Infinite cylindrical shell.*

External radius p, internal radius q.

The distribution of free magnetism will be similar to that on the solid cylinder, except that, as in the case of a spherical shell, the free magnetism on the interior surface will be of the opposite kind to that at corresponding points of the external surface.

For the external surface (red at North),

$$\mathrm{I}=\varkappa\mathrm{F}\frac{1+2\pi\varkappa\left(1-\frac{q^2}{p^2}\right)}{1+4\pi\varkappa+4\pi^2\varkappa^2\left(1-\frac{q^2}{p^2}\right)}.$$

Internal surface (blue at North),

$$I'=\varkappa F\frac{1}{1+4\pi\varkappa+4\pi^2\varkappa^2\left(1-\frac{q^2}{p^2}\right)}.$$

Hence for an external particle the coefficient will be $2\pi\left(I-\frac{q^2}{p^2}I'\right)$,

$$2\pi\left(I-I'\frac{q^2}{p^2}\right)=2\pi\varkappa F\frac{(1+2\pi\varkappa)\left(1-\frac{q^2}{p^2}\right)}{1+4\pi\varkappa+4\pi^2\varkappa^2\left(1-\frac{q^2}{p^2}\right)}$$

$$=\frac{2\pi\varkappa F}{2\pi\varkappa+1}\;\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{1}{2\pi\varkappa}}$$

nearly, when $\varkappa$ is large and $1-\frac{q}{p}$ small.

In the interior of the cylinder the coefficient is

$$-2\pi\{I-I'\}=-2\pi\varkappa F\frac{2\pi\varkappa\left(1-\frac{q^2}{p^2}\right)}{1+4\pi\varkappa+4\pi^2\varkappa^2\left(1-\frac{q^2}{p^2}\right)}$$

$$=-F\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{1}{2\pi\varkappa}}$$

nearly, if $\varkappa$ be large and $1-\frac{p}{q}$ be small; or whole force in interior

$$=\frac{F}{1+2\pi\varkappa\left(1-\frac{q}{p}\right)}.$$

Application to particular cases.

As we know from the general equations that the effect of any masses of soft iron may be represented by means of the coefficients a, b, c, d, e, f, g, h, k, and as we are in possession of formulæ which give the different parts of the deviation in terms of these coefficients, by far the most convenient mode of expressing the effect of any given mass of soft iron is to find the a, b, c, d, e, f, g, h, k to which it gives rise; and in what follows we shall suppose the formulæ involving these quantities and connecting them with the deviation-coefficients to be known.

Thus from the expressions we have given for the effect of a finite or infinite rod, we at once derive the coefficients a, b, c, they being the factors of X, Y, Z in the expressions for the force towards x, and so of the others. From these we might derive the coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, λ, χ; but there would be no interest in the general solution, as the rods we have to deal with in practice are always parallel to one of the principal axes, and these we shall therefore consider separately.

Transverse longitudinal masses of Iron extending from side to side as Iron beams.

Let m be the length of the beam, or in general the breadth of the vessel, r the distance of either end of the beam from the compass, S the area of the section of the beam. It is easily seen that such a beam will give no coefficient except

$$e=-\frac{\varkappa \mathrm{S} m}{r^3}.$$

Every such beam therefore diminishes the directive force and produces a + quadrantal deviation, the effect being directly proportional to the mass of the beam, inversely proportional to the cube of the distance of its ends.

If we have a rectangle of four beams, two fore-and-aft and two transverse, the compass being in or directly above or below the centre of the rectangle, l being the length of the two fore-and-aft beams, m of the two transverse beams, we shall have

$$a=-2\varkappa \mathrm{S}\frac{l}{r^3},$$

$$e=-2\varkappa \mathrm{S}\frac{m}{r^3},$$

whence

$$\lambda=1-\varkappa \mathrm{S}\,\frac{l+m}{r^3},$$

$$\mathfrak{D}=-\frac{\varkappa s}{\lambda}\,\frac{l-m}{r^3}.$$

Such beams may be compared to the armour-plating of a ship, and we thus see that for a compass near the centre of the ship, l being greater than m, the effect of such plating will be to diminish the quadrantal deviation.

In accordance with this result, we find that in the wood-built iron-plated ships, when the compasses are inside the rectangle of the armour-plating, the quadrantal deviation is very small.

When, as in the case of the Warrior and Black Prince, the plating does not extend from end to end, and the compasses are near or even outside one end, the case is different.

Thus if the fore-and-aft coordinates of the ends be x' and x, and the distances from the compass r' and r, we shall have

$$a=2\varkappa \mathrm{S}\left\{-\frac{x'}{r'^3}+\frac{x}{r^3}\right\},$$

$$e=2\varkappa \mathrm{S}\left\{-\frac{y}{r'^3}+\frac{y}{r^3}\right\},$$

$$\mathfrak{D}=\frac{\varkappa s}{\lambda}\left\{\frac{x+y}{r^3}-\frac{x'-y}{r'^3}\right\}.$$

When the plating extends abaft the compass x is negative, and when this is the case, x' being of course greater than y, so long as x is greater than y, or *so long as the plating*

extends half the breadth of the ship abaft the compass, it will diminish the quadrantal deviation.

$$\text{When } x=-\left\{y-(x'-y)\frac{r^3}{r'^3}\right\},$$

or *when the armour-plating extends a little less than half the breadth abaft the compass*, its effect on the quadrantal deviation vanishes, and when the distance is less than that last mentioned, it increases the quadrantal deviation.

If the central part of a beam be cut out, and if y and y' be the transverse coordinates, r, r' the distances of its outer and inner extremities from the compass,

$$e=2\varkappa\mathrm{S}\left\{\frac{-y}{r^3}+\frac{y'}{r'^3}\right\}.$$

Hence if such a beam be near the compass so that $\frac{y}{r^3}<\frac{y'}{r'^3}$, it will increase the directive force and diminish the quadrantal deviation; if distant it will have the opposite effect.

A vertical rod, z being the vertical coordinate of the upper, z' of the lower end, x and y being the horizontal coordinates, will produce

$$c=\varkappa\mathrm{S}x\left(\frac{1}{r^3}-\frac{1}{r'^3}\right),$$

$$f=\varkappa\mathrm{S}y\left(\frac{1}{r^3}-\frac{1}{r'^3}\right),$$

$$k=\varkappa\mathrm{S}\left(\frac{z}{r^3}-\frac{z'}{r'^3}\right).$$

The effect which is of most interest is that of k, as it affects the heeling error.

If z be negative, z' positive, or if the upper end of the beam be above and the lower end below the level of the compass, we see that k will be negative, and will in general diminish the heeling error.

If the rod be a short one of length n,

$$k=\frac{\varkappa\mathrm{S}n}{r^3}\left(3\frac{z^2}{r^2}-1\right);$$

here k will be $\pm$, as

$$\frac{z}{r}\gtrless\frac{1}{\sqrt{3}},$$

or, in other words, if the centre of the rod be within the cone traced out by a line through the compass, making an angle of 54° 45′ with the vertical, k will be positive, and the force of the rod will act downwards and increase the heeling error. On the other hand, if the centre of the rod be without the cone, k will be negative, and the force will act upwards and decrease the heeling error.

Hence we see that in all cases, except when the compass is raised very much above the upper part of the armour-plates, the effect of armour-plating will be to diminish the heeling error.

Thin Plate magnetized in its plane.

If the compass be above or below the centre of a rectangular plate, which may represent the iron deck of a ship, $2x$ being the length, $2y$ the breadth, n the thickness, z the height of the compass above it, r the distance from the compass to one corner, and v the volume of the plate,

$$a=-\frac{4\varkappa nxy}{r(x^2+z^2)}=-\frac{\varkappa v}{r}\cdot\frac{1}{x^2+y^2},$$

$$e=-\frac{4\varkappa nxy}{r.(y^2+z^2)}=-\frac{\varkappa v}{r}\,\frac{1}{y^2+z^2},$$

$$\mathfrak{D}=\frac{\varkappa v}{2r}\left\{\frac{1}{y^2+z^2}-\frac{1}{x^2+z^2}\right\},$$

or such a plate will always produce a diminution of the directive force, and if $x>y$, or if its length be in the fore-and-aft direction, a *positive* quadrantal deviation.

A vertical thin plate, such as a transverse bulkhead, may, as regards transverse induction, be considered as a series of thin horizontal beams giving a $-e$, diminishing λ and increasing $\mathfrak{D}$. As regards vertical induction, it may be considered as a series of vertical rods giving a $+c$ if before the compass, a $-c$ if abaft, and a $+k$ or $-k$ according nearly as the centres of the supposed vertical rods are within or without the cone we have described. There would be no difficulty in computing the effect of such a bulkhead of given position and thickness if $\varkappa$ were known.

Thick Plate magnetized perpendicularly.

If the length and breadth of the plate be infinite or very great compared to the distance of the compass, such a plate will produce no effect on the compass, the effect of one surface being exactly neutralized by that of the other.

When the dimensions of the plate are finite we may arrive at an approximate result, by supposing lines drawn from the compass to every point on the edge of the further surface. The parts of the two surfaces within the pyramid bounded by these lines will neutralize each other, leaving only a margin of the nearer surface to act on the compass. The effect of this may be easily computed, by computing the effect of four such red or blue lines, as the case may be, the free magnetism in a unit of length being

$$\frac{\mathrm{F}}{4\pi}\times \text{breadth of margin.}$$

From these considerations we see that the effect of even a thick armour-plating, magnetized perpendicularly, will not be great.

The effect of a thick transverse armour bulkhead, on a compass immediately above and near it, will be to produce a $-a$, which may be easily computed, as we may suppose the dimensions of the plate in every direction below its upper surface to be infinite.

If l be the thickness of the bulkhead, n the height of the compass above its centre,

$$a=-\frac{1}{2\pi}\,\frac{l}{n}.$$

Sphere.

Let the centre of the sphere be at a distance r from the centre of the compass, and let r make angles α, β, γ with the coordinate axes to head, to starboard, and to nadir, and let

$$\frac{\frac{4\pi}{3}\varkappa}{1+\frac{4\pi}{3}\varkappa}\frac{p^3}{r^3}=\mathrm{M}.$$

Then

$$\begin{aligned} a&=\mathrm{M}(3\cos^2\alpha-1), \\ b=d&=\mathrm{M}\ 3\cos\alpha\cos\beta, \\ c=g&=\mathrm{M}\ 3\cos\alpha\cos\gamma, \\ e&=\mathrm{M}(3\cos^2\beta-1), \\ f=h&=\mathrm{M}\ 3\cos\beta\cos\gamma, \\ k&=\mathrm{M}(3\cos^2\gamma-1), \end{aligned}$$

whence

$$\begin{aligned} \lambda&=1+\frac{\mathrm{M}}{2}\{1-3\cos^2\gamma\}, \\ \mathfrak{A}&=0, \\ \mathfrak{B}&=\frac{\mathrm{M}}{\lambda}3\cos\alpha\cos\gamma\tan\theta, \\ \mathfrak{C}&=\frac{\mathrm{M}}{\lambda}3\cos\beta\cos\gamma\tan\theta. \\ \mathfrak{D}&=\frac{\mathrm{M}}{\lambda}\cdot\frac{3}{2}(\cos^2\alpha-\cos^2\beta), \\ \mathfrak{E}&=\frac{\mathrm{M}}{\lambda}3\cos\alpha\cos\beta, \end{aligned}$$

From these we see that a sphere, wherever placed, will increase λ and give a $-k$ if

$$\cos\gamma<\frac{1}{\sqrt{3}}$$

or

$$\gamma>54°\ 45',$$

and will decrease λ and give a $+k$ if $\gamma<54°\ 45'$.

Hence if, as before, we suppose a double cone traced out by a line passing through the compass, making an angle $54°\ 45'$ with the vertical, all spherical masses of iron whose centres are placed without the cone will increase the directive force and diminish the usual heeling error. All spherical masses whose centres are placed within the cone will diminish the directive force and increase the heeling error. Hence, as far as possible, no iron should be either below or above the compass within an angle of $54°\ 45'$ of the vertical passing through the compass.

If $\cos\alpha>\cos\beta$, or if the centre of the sphere be in either fore-and-aft quadrant, the

effect of the sphere is to increase the quadrantal deviation; if in the starboard or port quadrant, it will decrease the quadrantal deviation.

If we have two spheres, one on each side and at the level of the compass, $\alpha=90°$, $\gamma=90°$, $\beta=0°$ and $180°$, whence

$$\lambda=1+\mathrm{M},$$

$$\mathfrak{D}=-\frac{3\mathrm{M}}{1+\mathrm{M}}=\frac{3}{1+\left(\frac{r}{p}\right)^3}\text{ nearly.}$$

Hence we get the following for the effect of two such spheres according to the number of semidiameters which their centres are distant from the centre of the compass.

r.	$\mathfrak{E}$.	D.
$2p$	·333	19° 30′
$3p$	·107	6 10
$4p$	·046	2 40
$5p$	·023	1 20

Hence also we find the distance of the spheres required to correct any given quadrantal deviation $\mathfrak{D}$,

$$r=p\sqrt[3]{\frac{3}{\mathfrak{D}}-1}.$$

As we have supposed $\dfrac{\frac{4\pi}{3}\kappa}{1+\frac{4\pi}{3}\kappa}=1$, the deviation which two balls of iron of the usual kind will correct will be one or two per cent. less than the above.

When the sphere is in either of the diagonal planes, $\alpha=45°$, $\beta=45°$, or $\alpha=-45°$, $\beta=135°$,

$$\mathfrak{D}=0,\text{ and }\mathfrak{E}=\pm\frac{3}{2}\frac{\mathrm{M}}{\lambda},$$

or $\mathfrak{E}$ is the same as the $\mathfrak{D}$ when the sphere is in a principal plane. This we should of course anticipate.

From the expression $\mathfrak{B}=\frac{\mathrm{M}}{\lambda}\,3\cos\alpha\cos\gamma\tan\theta$, we see that in the northern hemisphere, if the sphere be below and before, or above and abaft the compass, we have a + semicircular deviation; if above and before, or below and abaft, a − semicircular deviation.

Spherical Shell.

The effect, if the compass be exterior to the shell, will be precisely the same as that of a sphere if for M we substitute

$$\mathrm{M}\,\frac{\left(1+\frac{4}{3}\pi\kappa\right)\left(1+\frac{8}{3}\pi\kappa\right)\left(1-\frac{q^3}{p^3}\right)}{1+4\pi\kappa+\frac{4}{3}\pi\kappa\,\frac{8}{3}\pi\kappa\left(1-\frac{q^3}{p^3}\right)},$$

or nearly, when κ is large and $1-\frac{q}{p}$ small,

$$M\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{3}{8\pi\kappa}}.$$

Hence we see that the force of the shell will be half that of a sphere of equal external radius if κ be 12 and the thickness of the shell be $\frac{1}{100}$ of the semidiameter, or if $\kappa=24$ and the thickness be $\frac{1}{200}$ of the semidiameter, or if $\kappa=36$ and the thickness of the shell be $\frac{1}{300}$ of the semidiameter.

Hence the effect of a tank $\frac{1}{10}$th of an inch thick and 4 feet diameter would probably be about one-third that of a solid mass of the same dimensions.

The effect of such a mass as a rifle-tower $4\frac{1}{2}$ inches thick and 10 feet in diameter will be nearly the same as if it were of solid iron. Such a tower placed in front of a compass, as in the Warrior, will give a considerable $+a$, a $-e$ of half the amount, and therefore increase λ and $\mathfrak{D}$, and if the compass be neither much above nor below it, decrease the heeling error.

Infinite cylinder magnetized perpendicularly to its length.

A compass placed at a considerable height above the deck, near an iron mast or funnel, may be considered as acted on by a vertical cylinder or cylindrical shell of infinite length. If r be the distance of its centre from the centre of the compass, p and q the radii of the outer and inner surfaces of the cylinder, then when the cylinder is solid,

$$M=\frac{2\pi\kappa}{1+2\pi\kappa}\frac{p^2}{r^2}$$

and when the cylinder is hollow

$$M=\frac{2\pi\kappa}{1+2\pi\kappa}\frac{p^2}{r^2}\frac{(1+2\pi\kappa)^2\left(1-\frac{q^2}{p^2}\right)}{1+4\pi\kappa+4\pi^2\kappa^2\left(1-\frac{q^2}{p^2}\right)}$$

$$=\frac{2\pi\kappa}{1+2\pi\kappa}\frac{p^2}{r^2}\frac{1-\frac{q}{p}}{1-\frac{q}{p}+\frac{1}{2\pi\kappa}}$$

nearly, if κ is large and $1-\frac{q}{p}$ small.

Also

$$a = M,$$
$$e = -M;$$

hence

$$\lambda = 1,$$
$$\mathfrak{D} = M;$$

whence we get the remarkable result, that a long vertical cylinder or a cylindrical shell

does not alter the mean directive force on a compass placed near its centre as regards elevation.

It may be interesting to compare the effect of two solid stanchions placed one on each side of the compass with that of two solid spheres, in correcting the quadrantal deviation. The effect of the stanchions would be nearly

$$2\frac{p^2}{r^2},$$

whence

r.	$\mathfrak{D}$.	D.
		° ′
$2p$	·500	30 0
$3p$	·222	12 50
$4p$	·125	7 10
$5p$	·080	4 36

A mast or stanchion placed as we have supposed would generally diminish the heeling error.

We may compare the effect on the directive force of a compass on the main deck of an iron ship with the effect on a compass in the interior of a spherical shell.

In some ships the value of λ at the main-deck compass is about ·75.

Comparing this value with the expression for the force in the interior of a spherical shell, viz.,

$$\frac{F}{1+\frac{8\pi\varkappa}{3}\left(1-\frac{q}{p}\right)},$$

we have

$$\frac{1}{\cdot 75}=\frac{4}{3}=1+\frac{8\pi}{3}\varkappa\left(1-\frac{q}{p}\right),$$

or

$$1-\frac{q}{p}=\frac{1}{8\pi\varkappa};$$

taking $\varkappa$ as 24,

$$1-\frac{q}{p}=\frac{1}{600}$$

nearly, or the effect is the same as if the compass were inclosed in a spherical shell of an inch thick and 50 feet radius, or half an inch thick and 25 feet radius.

We may observe that at present one of the great difficulties in deducing numerical results as to the effect of rods or plates of iron, arises from our ignorance of the value of $\varkappa$ for iron used for building or plating ships. We hope to be able on some future occasion to be able to communicate to the Royal Society the result of observations made for the purpose of determining this value in plates of iron of different kinds.

GENERAL CONCLUSIONS.

The following appear to be the principal conclusions to be drawn from the application of observation and theory to the magnetic phenomena in iron ships.

1. The original semicircular deviation depends principally on the direction of the ship's head in building, and consists principally in an attraction of the north point of the needle to the part of the ship which was (nearly) south in building.

2. This attraction is caused by the subpermanent magnetism induced in the ship when building, by the horizontal force of the earth.

3. If we consider separately, first, the effect of the subpermanent magnetism induced by the fore-and-aft component of the horizontal force, and secondly, the effect of the subpermanent magnetism induced by the transverse component of the horizontal force, the first is relatively less than the second. This, if the direction of the ship in building does not coincide with a cardinal point, modifies the direction of the semicircular deviation produced.

4. A third part, being the remainder of the semicircular deviation, is independent of the direction of the ship in building. It is the effect of the subpermanent and transient magnetism induced in the ship by the vertical force of the earth, and it consists in an attraction of the north point of the needle to the bow or stern.

In the usual place of the Standard Compass this part is, in the northern hemisphere, an attraction of the north point of the needle towards the bow; but if the compass is placed nearly in front of a large vertical mass of iron, as the stern-post, it may be towards the stern.

5. The first and second parts of the semicircular deviation diminish rapidly after the ship has been launched, the second generally most rapidly; but after a time, which may be taken roughly as a year, if the ship has been allowed to swing on all azimuths, they attain a very fixed and permanent amount, from which they do not afterwards vary to any great extent.

The third part changes little, if at all, so long as the ship remains in the same latitude.

6. The changes which take place in the semicircular deviation of a ship built East and West are generally relatively greater than in one built North and South.

7. The transient magnetism induced by the earth's horizontal force adds to the effect of the subpermanent magnetism induced by the same force, when she is on the stocks, and afterwards when her head is in the same direction in which it was while building.

8. The effect of the subpermanent and transient magnetism induced by the horizontal force when the ship is on the stocks is principally, and if the ship is built on a cardinal point entirely, to produce a diminution of the directive force on the needle, and very little, and if built on a cardinal point not at all, to produce deviation.

9. The same effect (nearly) is produced at a subsequent time if the ship's head is placed on the direction in which it was while building.

10. This diminution of the directive force is greater if the ship has been built East and West than if built North and South.

11. The deviations in an iron ship which has been built East or West are more prejudicial than in a ship built North or South in the following respects:—

1. They are less symmetrical and regular, and therefore more perplexing to the seaman.
2. They change more relatively after launching.
3. They diminish the directive force more when the ship is on particular points.

12. When a ship has been built head North, the upper part of the stern and the lower part of the bow are strongly magnetized; the upper part of the bow and the lower part of the stern are weakly magnetized. When a ship has been built head South, the upper part of the bow and the lower part of the stern are strongly, the upper part of the stern and the lower part of the bow are weakly magnetized.

Consequently in ships built head North, a compass placed near the stern will have a large semicircular deviation.

13. In the last case there will be a large downward force on the north point of the needle, which will produce a large heeling error. In ships built head South, both the last errors will probably be small.

14. On the whole, for compasses to be placed in the after part of the ship, the best direction for building is head South. For compasses near the centre of the ship, the directions head North and head South are nearly equally good.

15. The diminution of the mean directive force is the mean of the diminution caused by the transient magnetism induced by the horizontal force when the ship's head is North or South, and that induced when her head is East or West, *i. e.* it is the mean of the thrust from the north end and from the north side.

16. The quadrantal deviation is caused by the excess of the latter over the former, *i. e.*, by the excess of the thrust from the north side over the thrust from the north end.

17. The diminution of the directive force and the amount of the quadrantal deviation are nearly the same at the same level in different parts of the ship. They increase in descending from the position of the Standard Compass to the compasses on the upper and main decks. They diminish with the lapse of time.

18. By substituting wood for iron in the part of the deck below and above the compass, and within an angle of 35° 15′ of the vertical line passing through the compass, and having no masses of iron with their centres within 54° 15′ of the same vertical line, the directive force is increased and the quadrantal and heeling error generally diminished.

19. In selecting a place for the Standard Compass, care should be taken to avoid as much as possible the proximity of the ends of elongated masses of iron, particularly if placed vertically; or, if they cannot be avoided, then a place should be selected where they diminish instead of increasing the semicircular deviation.

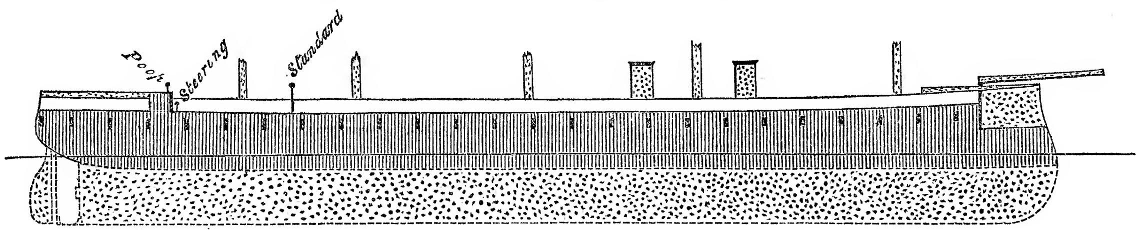
The neighbourhood of rifle and gun turrets in ships carrying them should be as much as possible avoided.

20. In the construction of iron-built and iron-plated ships, regard should be had to the providing a suitable place for the Standard Compass. It is not difficult for any one who has studied the question, to suggest arrangements which would greatly mitigate the injurious effects of the iron of the ship; the difficulty is to reconcile them with the requirements of construction and of working.

Postscript.

Since the foregoing paper was read, additional observations of deviations have been made in the Achilles and Defence, and observations in two new iron-built armour-plated ships, the Minotaur and Scorpion, the results of which are contained in the annexed Table. The observations in the Achilles show a continued diminution in the value of 𝔅 and a continued tendency in ℭ to return to its original value. The Defence continues to show great permanence both in 𝔅 and ℭ.

The Minotaur, of which it has been thought desirable to give a woodcut drawn to



the same scale as the ships represented in Plate XI., illustrates in a very remarkable manner some of the principles deduced from other ships. The Minotaur is the first iron-built ship completely plated from end to end; her quadrantal deviation is consequently small. Having been built and plated head north, the original deviations in all the compasses were very large. In the steering and poop compasses the maximum deviation was above 60°. With deviations of this amount the compass becomes useless unless corrected by magnets, and magnets were consequently applied, which removed almost entirely the semicircular deviation. Probably in a very short time we shall find the original $-$𝔅 of these compasses to have so far diminished that the compasses will be found to be greatly over corrected and to have a considerable $+$𝔅. Magnets were also applied to the Standard Compass. The heeling error at the poop compass is very large, 2° 46′. This arises from the compass being so near the stern of the ship, built and plated head north, and also from its being elevated above the armour-plating. It is interesting to contrast it with the heeling error of the steering compass, where from the peculiar configuration of the armour-plating being such as to give a $-k$, the heeling error is diminished and of a moderate amount.

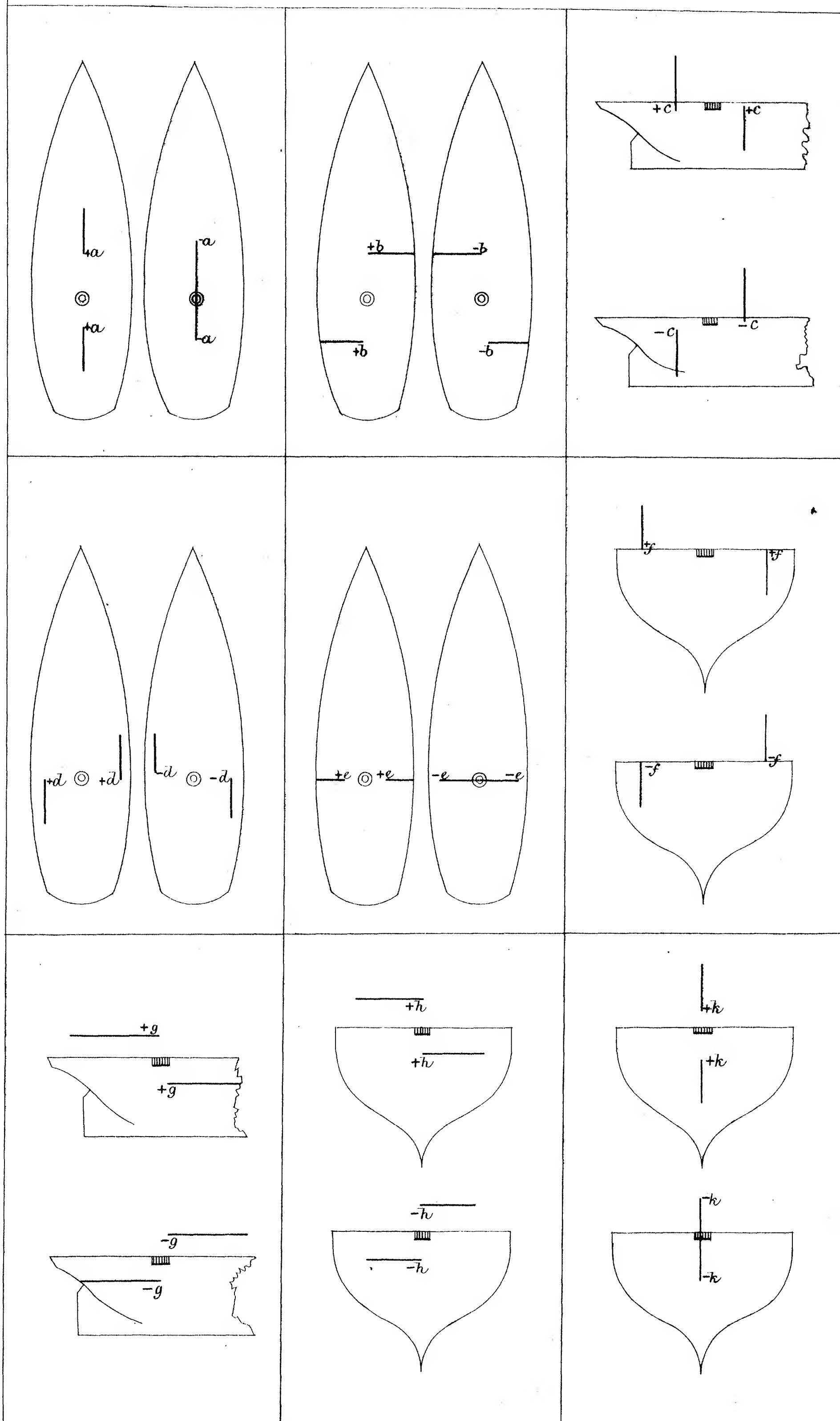
The Scorpion is a remarkable instance of the change which takes place in the semicircular deviation from a change of position in a new iron-built vessel. Having been built head N. 76° W., or S. 254° E., the original value of 𝔅 was $-\cdot 246$, and the original starboard angle was $233\frac{1}{2}$°. After lying four months head S. 47° W. or S. 313° E., the value of 𝔅 changed its sign and became $+\cdot 225$, and the starboard angle increased to $303\frac{1}{2}$°, thus following very nearly the direction of the south line in the ship. The Scorpion is an instance of the successful correction of the heeling error by means of a vertical magnet. This reduced the heeling error from 1° 38′ to 2′ for each degree of heel.

Ship.	Compass.	Place.	Date.	Approximate coefficients. A	B	C	D	E	Exact coefficients. 𝔄	𝔅	ℭ	𝔇	𝔈
				° ′	° ′	° ′	° ′	° ′					
MINOTAUR. (6621 tons), 1350 horse-power, 26 guns, Iron-cased, iron hull.	Standard.	Victoria Docks,	March 28, 1865	*By deviation and horizontal force on one point:* λ *and* D *of March 30 adopted*						−·487	+·174	...	...
		River Thames,	March 30, 1865 ...	−0 47	−23 26	+ 6 4	+ 5 41	−0 54	−·014	−·420	+·099	+·100	−·016
		Sheerness	April 10, 1865 ..	−0 5	−20 30	+ 4 25	+ 5 43	−0 26	−·001	−·379	+·069	+·100	−·007
				−0 5	+ 0 44	− 0 16	+ 5 43	−0 26	*after correction by magnets.*				
Built on same slip as Warrior; head N. 3° E. magnetic. Launched Dec. 12, 1863.	Starboard steering.	Sheerness	April 10, 1865 ..	......	−61 0	+ 0 45	......	......	+·009	−·965	+·015	+·103	−·002
				+0 32	− 0 28	+ 2 8	+ 5 56	−0 7	*after correction by magnets.*				
Plated head N. 22° E. in Victoria Docks.	Poop (on fore part),	Sheerness	April 10, 1865 ..	......	−60 0	+ 2 0	......	......	+·022	−·948	+·038	+·086	−·001
				+1 16	− 1 55	+ 5 8	+ 4 55	−0 5	*after correction by magnets.*				
SCORPION. (1857 tons), 350 horse-power, 4 guns, Iron-cased turret ship, iron hull.	Standard.	Birkenhead ...	October 31, 1864	*By deviation and horizontal force on one point:* λ *and* D *of March 1865 adopted, with small allowance for lapse of time*						−·246	−·355	+·190	*assumed.*
Built at Birkenhead; head N. 76° W.		Birkenhead	March 14, 1865	*From observations made in one quadrant after ship had been lying four months S. 47° W.*						+·225	−·341	+·180	...
		Birkenhead	March 15, 1865	−0 53	+ 0 32	+ 1 43	+10 47	−0 52	−·015	+·009	+·030	+·187	−·015
				after correction by magnets.									
ACHILLES (continued).	Standard.	Portland	April 1865	......	+16 50	+12 30	+ 6 40	......	...	+·322	+·191	+·115	...
		Lisbon	*May* 4, 1865	......	......	......	......	......	...	+·274	+·132	...	...
DEFENCE (continued).	Standard.	Portland	April 3, 1865	+0 13	+20 19	− 0 14	+ 6 09	−0 36	+·004	+·367	−·004	+·107	−·010
		Lisbon	*May* 1, 1865	+0 23	+16 51	− 1 15	+ 6 16	+0 04	+·007	+·307	−·021	+·109	+·001

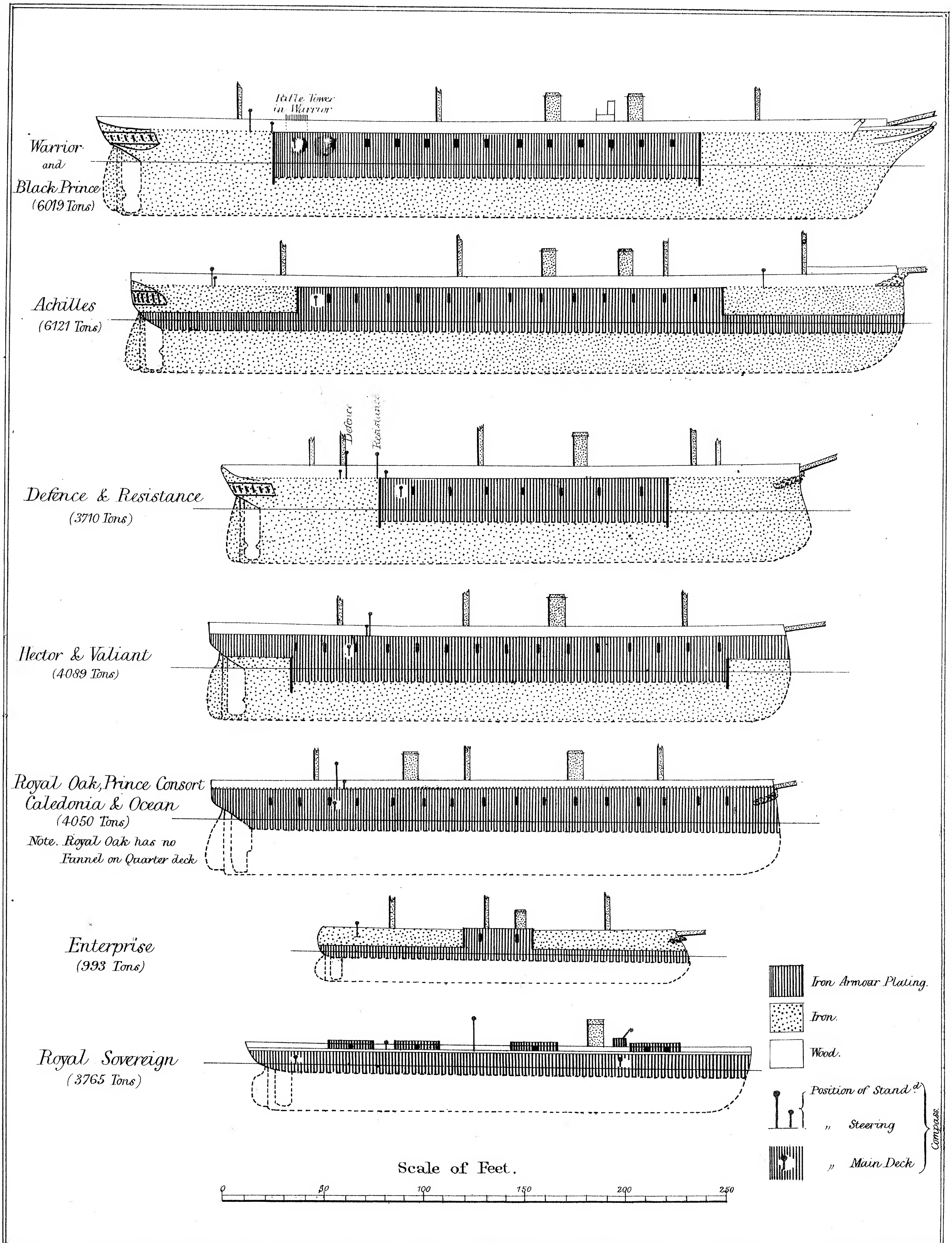
Maximum of semicircular deviation $\sqrt{B^2+C^2}$	Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$ *.		Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction.		Part of D from		Mean Vertical force, μ ‡	Heeling coefficient to *windward*, χ	Heeling coefficients from		$\frac{g}{\tan\theta}$ ‡	g †
	Amount.	Direction.			Fore-and-aft, a †	Transverse e †	Fore-and-aft induction.	Transverse induction.			Vertical induction in transverse iron.	Vertical force and induction in vertical iron.		
°		°					° ′	° ′		° ′	° ′	° ′		
...	·516	160½												
24¼	·432	166½	·876	1·142	−·036	−·212	−1 12	+ 6 57						
21¼	·385	169½	·892	1·121	−·019	−·197	−0 38	+ 6 51	1·442	+1 21	+0 35	+0 46		
61	·965	179	·811	1·233	−·106	−·272	−3 43	+ 9 42	1·091	+1 7	+0 50	+0 17		
60¼	·950	177¾	·826	1·211	−·103	−·245	−3 33	+ 8 30	1·660	+2 46	+0 46	+2 0		
......	·434	233½	·810 *assumed.*		......	......	......	......	1·472	+1 39				
......	·406	303½	......	......	......	......	......	......	1·636	+1 38	+1 02	+0 36	−·050	
......	......	......	·838	1·193	−·037	−·350	−0 7	+10 57	{·826 *after correction by vertical magnet.*	+0 2				
21	·374	30½	·844	1·185	−·059	−·253	−1 57	+ 8 38						
......	{·306 ·384}	26	·820	1·219	−·086	−·274	−3 2	+ 9 37						
20⅓	·367	359½	·875	1·143	−·031	−·219	−1 02	+ 7 13						
16¾	{·308 ·387}	356	·855	1·169	−·052	−·238	−1 46	+ 8 0						

* Mean force to North (λH) being unit. † Earth's Horizontal force (H) being unit. ‡ Earth's Vertical force (Z) being unit.

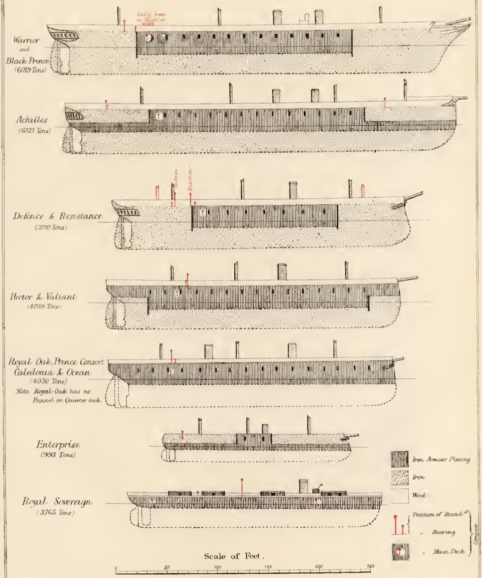
Diagram showing the positions of the nine soft iron rods which represent the whole of the soft iron of a ship as regards its action on the compass.



J. Basire, lith.



J. Basire lith.



Rifle boxes in Warrior
Warrior and Black Prince (6019 Tons)
Achilles (6121 Tons)
Defence
Resistance
Defence & Resistance (3710 Tons)
Hector & Valiant (4089 Tons)
Royal Oak, Prince Consort Caledonia & Ocean (4050 Tons)
Note Royal Oak has no Funnel on Quarter deck
Enterprise (993 Tons)
Royal Sovereign (3765 Tons)
Iron Armour Plating
Iron
Wood
Position of Stand^d
" Steering
" Main Deck
Compass
Scale of Feet.
0 50 100 150 200 250